GPS Range and Range Difference Sigmas*

Jim Wright Bill Chuba

24 November 2003

1 Introduction

There are several ways in which GPS range measurements can be differenced. The first purpose herein is to present equations for calculating the white noise sigma value to be used in the Orbit Determination Tool Kit (ODTK) sequential filter for range difference white-noise, given a sigma value on the original range white-noise sequence. Second, a short discussion is presented to enable the ODTK user to adopt, or derive, a sigma value on the original range white-noise sequence.

The simulation of each GPS range measurement requires the specification of a range white-noise sigma. Range differences are formed from different NAVSTARs to remove USER spacecraft clock effects. USER spacecraft two-frequency range differences are formed from the same NAVSTAR to remove ionospheric effects.

When a range difference is formed from two range measurements with the same frequency, but from different NAVSTARs, there derives an increased white-noise sigma on the range difference that must be accounted for by the filter. Also, when a range weighted-difference is formed for two-frequency ranging to remove first-order ionosphere effects, there derives an increased white-noise sigma on the weighted difference that must be accounted for by the filter. Further, if multi-channel range weighted-differences are differenced again to remove USER clock effects, then there derives increased white-noise sigma on these differences of differences.

1.1 Initial Notation and Definitions

Denote a GPS range measurement with ρ_f^n, where $n \in \{1, 2, 3, \ldots, 28\}$ identifies a particular GPS NAVSTAR spacecraft, and where $f \in \{L_1, L_2\}$ identifies frequency, either L_1 or L_2.

1.1.1 Range Difference (Remove Clock Effects)

Define range difference ρ_f:

$$\rho_f = \rho_f^i - \rho_f^j, \qquad i, j \in \{1, 2, 3, \ldots, 28\} \tag{1}$$

a first range difference used to remove USER clock effects from range measurements with the same frequency f, but from different NAVSTARs i and j.

1.1.2 Weighted Range Difference (Remove Ionospheric Effects)

Define range difference ρ^n:

$$\rho^n = \frac{\alpha}{\alpha - 1}\rho_1^n - \frac{1}{\alpha - 1}\rho_2^n \tag{2}$$

the weighted range difference for removal of ionosphere[1] from range measurements with different frequencies L_1 and L_2, but from the same NAVSTAR n, where:

[1] GPS ICD 200

$$\alpha = \left(\frac{77}{60}\right)^2 = 1.6469444\ldots \tag{3}$$

Second Range Difference (Remove Clock Effects) Define range difference ρ:

$$\rho = \rho^i - \rho^j, \qquad i, j \in \{1, 2, 3, \ldots, 28\} \tag{4}$$

a second range difference used to remove USER clock effects from weighted range differences.

2 Formulae for Filter Sigmas on GPS Range Differences

2.1 Difference C/A Code Range Measurements (Remove USER Clock Effects)

For C/A Code ($f = L_1$) range measurements from distinct NAVSTARs, let σ_f^i denote the GPS range sigma, and let σ_f denote the filter sigma on C/A Code range differences to remove USER clock effects. Then:

$$\sigma_f = \left[\sqrt{2}\right] \sigma_f^i \approx [1.4142136]\, \sigma_f^i$$

2.2 Difference P Code Range Measurements (Remove Ionospheric Effects)

For P Code ($f \in \{L_1, L_2\}$) range measurements from the same NAVSTAR, let σ_1^n denote the GPS range sigma, and let σ^n denote the filter sigma on P-Code range differences to remove ionospheric effects. Then:

$$\sigma^n = \left[\frac{\sqrt{\alpha^2 + 1}}{\alpha - 1}\right] \sigma_1^n \approx [2.9782553]\, \sigma_1^n$$

2.2.1 Difference P Code Differences (Remove USER Clock Effects)

For P Code range differences from distinct NAVSTARs, let σ_1^i denote the GPS range sigma, and let σ denote the filter sigma on differences[2] of P-Code range differences[3]. Then:

$$\sigma = \left[\frac{\sqrt{2\left(\alpha^2 + 1\right)}}{\alpha - 1}\right] \sigma_1^i \approx [4.211889]\, \sigma_1^i$$

3 Default Values for GPS Range Sigmas

3.1 Difference C/A Code Range Measurements (Remove USER Clock Effects)

Default value:

$$\sigma_f^i = \sigma_f^j = 3m$$

Then:

$$\sigma_f = \left[\sqrt{2}\right] \sigma_f^i \approx [1.4142136]\, \sigma_f^i = 4.2426407m$$

See Fig. 1 for a CHAMP C/A-Code example using these values for range white noise sigmas. The ionosphere range sigma was defined to be one-fourth of the modeled value of ionosphere range.

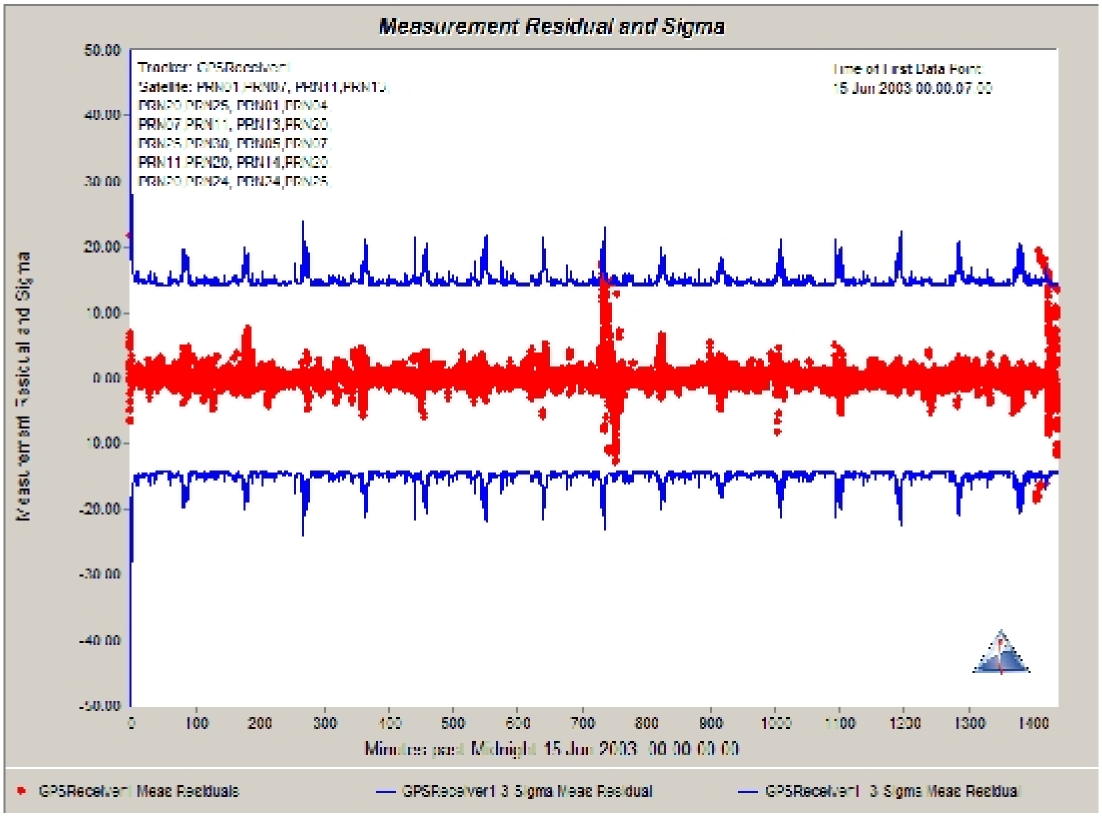

Figure 1: CHAMP C/A Code Range Difference Residuals (CA SD)

Other Sigma Values	Description C/A Code
$\rho_{IONO}/4$	ionosphere sigma
$1m$	NAVSTAR Radial
$2m$	NAVSTAR Intrack
$3ns$	NAVSTAR Clocks

Table 1: Other Sigma Values (C/A Code)

3.2 Difference P Code Range Measurements (Remove Ionospheric Effects)

Default value:

$$\sigma_1^n = \sigma_2^n = 1m$$

Then:

$$\sigma^n = \left[\frac{\sqrt{\alpha^2 + 1}}{\alpha - 1} \right] \sigma_1^n \approx [2.9782553]\, \sigma_1^n = 2.9782553m$$

[2]Remove USER clock effects.
[3]Remove ionosphere effects.

3.3 Difference P Code Differences (Remove USER Clock Effects)

Default value:

$$\sigma_1^i = \sigma_1^j = \sigma_2^i = \sigma_2^j = 1m$$

Then:

$$\sigma = \left[\frac{\sqrt{2\left(\alpha^2 + 1\right)}}{\alpha - 1} \right] \sigma_1^i \approx [4.211889]\, \sigma_1^i = 4.211889m$$

See Fig. 2 for a CHAMP P-Code example using these values for range white noise sigmas.

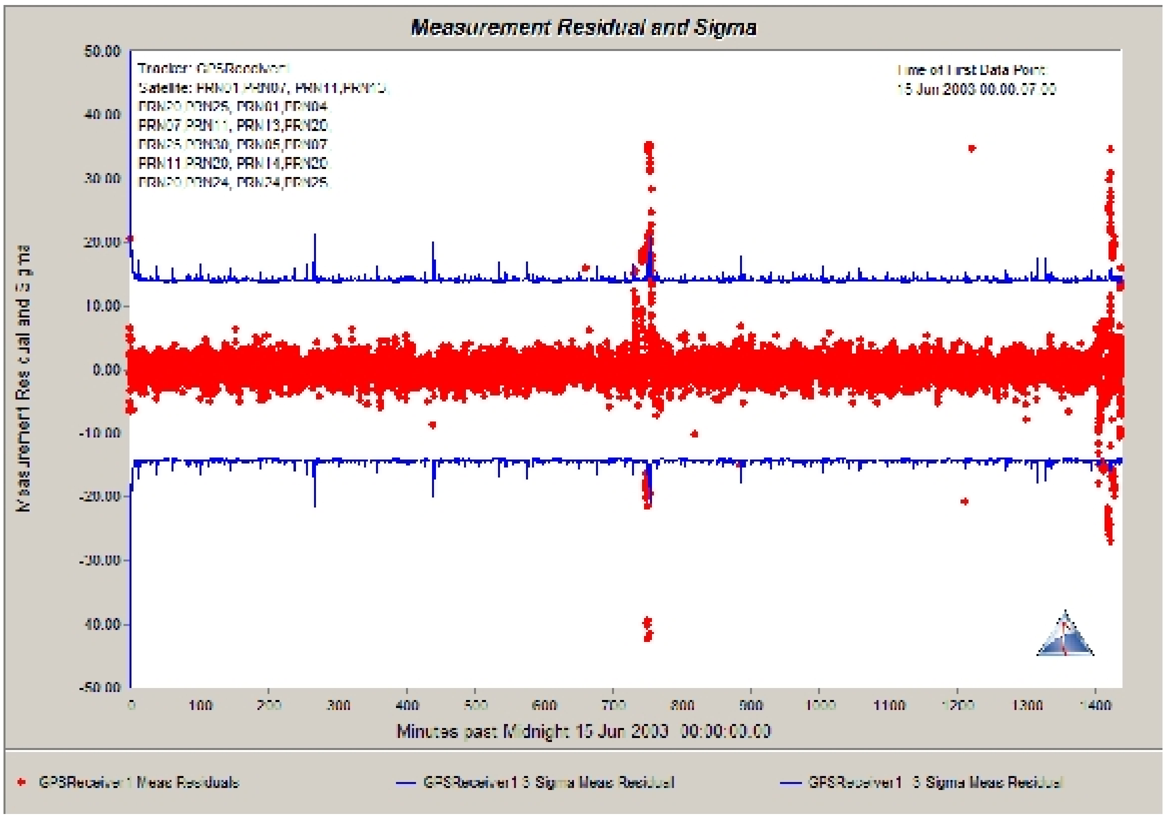

Figure 2: CHAMP P-Code Range Difference Residuals (DF SD)

Other Sigma Values	Description P-Code
$1m$	NAVSTAR Radial
$2m$	NAVSTAR Intrack
$3ns$	NAVSTAR Clocks

Table 2: Other Sigma Values (P-Code)

4 GPS Range White Noise Sigma Value

Given a range white noise sigma value, we show calculations herein for values of derived white noise sigmas on various range differences. But where should the ODTK user obtain a useful value for a range white noise sigma to begin with?

GPS receiver builders may not know the values of associated range white noise sigmas for their receivers. There is an inherent difficulty in isolating receiver white noise from receiver serially-correlated noise. All receivers have both. The test equipment, and appropriate testing, required is very expensive, even for ground-based bench testing. If the receiver were tested in the space environment, the white noise signal would be further masked by environmental serially-correlated noise, and the range sigma value calibration result for least squares estimation would be significantly different than for the use of an optimal sequential filter-smoother.

The ODTK default values presented below were tested against CHAMP C/A Code and P-Code data, and range difference residual graphics were generated.

In order to obtain an optimal estimate of an appropriate receiver-dependent range white noise sigma value, one should:

- Obtain an appropriate set of GPS range data for the GPS receiver of interest
- Perform a calibration study on the GPS range data using ODTK to isolate white noise

5 White Noise

This note is about Gaussian white noise sequences. So what is a white noise sequence?

5.1 White Noise Sequence Definitions

5.1.1 Mutual Independence

Any sequence of events that are mutually independent is a white noise sequence[4].

5.1.2 Random Walk Derivative

A sequentially differenced random walk sequence is a white noise sequence[2].

5.2 ODTK Simulation

5.2.1 Linear Congruential Generator

For applications other than clocks we invoke a linear congruential generator[5] to produce a white sequence of pseudo random numbers with uniform distribution and zero mean on the interval $(0, 1)$. The uniform sequence is then transformed to a Gaussian white sequence with zero mean and unit variance.

5.2.2 Differenced Random Walk

For clocks[1] we construct a zero-mean Gaussian random walk sequence in time, then we form sequential ratios of random walk sequence differences divided by their time differences to construct a zero-mean Gaussian white noise sequence. This enables a physically correct relation between clock phase (time) and clock frequency (the generalized derivative of phase).

5.3 Physical Sources

Physically, thermal noise is created in every electronic circuit by its resistivity to flow of electrons – resistance generates heat. Thermal noise is Gaussian white noise[3]. Thus transmitters, receivers, transceivers, and transponders all create Gaussian white noise. Range measurements, generated by transmitters and receivers, have additive Gaussian white noise components.

5.4 Optimal Sequential Orbit Determination

With the state estimate structure defined correctly, all non-white signals in the range measurements are absorbed by state estimates, so that range residuals are white.

References

[1] Allan, D.W., *Time and Frequency Characterization, Estimation, and Prediction of Precision Clocks and Oscillators*, NIST Technical Note 1337, 1990

[2] Bucy, Richard S., Joseph, Peter D., *Filtering for Stochastic Processes with Applications to Guidance*, Interscience Publishers, New York, 1968

[3] Davenport, Wilbur B., Root, William L., *Random Signals and Noise*, McGraw-Hill, New York, 1958

[4] Jazwinski, Andrew H., *Stochastic Processes and Filtering Theory*, Academic Press, New York, 1970

[5] Press, William H., Teukolsky, Saul A., Vetterling, William T., Flannery, Brian P., *Numerical Recipes in C*, Cambride U. Press, N.Y., 1988