Attitude Uncertainty OpNav Deweighting

Nick Stankey

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The following is the derivation that shows how to convert attitude uncertainty in a three-component representation to quaternion uncertainty. Start with the three-component Euler Axis/Angle attitude representation and the definition of a quaternion.

$$\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3]^T \tag{1}$$

$$\theta = |\boldsymbol{\theta}| \tag{2}$$

$$q = [e_1 sin(\frac{\theta}{2}), e_2 sin(\frac{\theta}{2}), e_3 sin(\frac{\theta}{2}), cos(\frac{\theta}{2})] \tag{3}$$

Then apply the small angles approximation to create an error quaternion.

$$\delta q \approx [e_1 \frac{\theta}{2}, e_2 \frac{\theta}{2}, e_3 \frac{\theta}{2}, 1] \tag{4}$$

$$\delta q \approx [\frac{\theta_1}{\theta}\frac{\theta}{2}, \frac{\theta_2}{\theta}\frac{\theta}{2}, \frac{\theta_3}{\theta}\frac{\theta}{2}, 1] \tag{5}$$

$$\delta q \approx [\frac{\theta_1}{2}, \frac{\theta_2}{2}, \frac{\theta_3}{2}, 1] \tag{6}$$

$$P_{\delta q} = E[\delta q \delta q^T] = \begin{bmatrix} & & & 0 \\ & \frac{1}{4}P_\theta & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

P_θ is specified by the user in the ODTK application and represents the 3x3 covariance of the three-component attitude error vector. This can be specified in the white panel as arc seconds, where the diagonals are equal to the square of the value entered. The full covariance matrix can otherwise be specified directly as a variance in an attitude file, with its units as arc seconds^2.

The uncertainty is then mapped to the inertial pointing direction.

$$P_I = \frac{\partial M}{\partial q} \rho_s E[\delta q \delta q^T] \rho_s^T \frac{\partial M}{\partial q}^T \tag{8}$$

M represents the spacecraft body frame attitude with respect to inertial, ρ_s is the unit vector of the OpNav measurement direction coordinated in the spacecraft body frame, q represents the spacecraft body frame attitude quaternion, and $E[\delta q \delta q^T]$ is the attitude uncertainty represented in quaternion space.

Next, the uncertainty is mapped to right-ascension and declination space when computing OpNav point or limb measurements. If deweighting is enabled, this uncertainty is then added to the measurement covariance when processing the measurement update.