



## **USING EOP AND SPACE WEATHER DATA FOR SATELLITE OPERATIONS**

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# **15<sup>th</sup> AAS/AIAA Astrodynamics Specialist Conference**

**LAKE TAHOE, CALIFORNIA, AUGUST 7-11, 2005**

**AAS Publications Office, P.O. Box 28130, San Diego, CA 92198**



## Using EOP and Space Weather Data For Satellite Operations

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Since the demonstration of the first numerically generated space catalog by the United States Navy in 1997, a new issue exists of how to transition from the two-line element sets (TLEs) to routine use of numerical vectors in satellite flight dynamics operations. This generates some unique challenges. The historical TLE operations used analytically generated datasets which either ignored or approximated the precise coordinate system and force models required to accurately model the satellite. Numerical operations require precise adherence to a coordinate system and specific force models. In particular, the Earth Orientation Parameter (EOP) and space weather data are crucial to proper calculations using numerically generated state vectors. This paper investigates the available data, compares products within and between organizations, and provides a methodology for which data should be used for satellite operations.

### INTRODUCTION

The use of numerically generated state vectors for satellite operations is not new to astrodynamics. However, with the first numerically generated space catalog by the U.S. Navy in 1997 (Coffey and Neal, 1998), the potential to replace the existing TLEs with numerical results is feasible. This poses some unique challenges for the astrodynamics community. The historical TLE operations used analytically generated datasets which either ignored or approximated the precise coordinate system and force models required to accurately model the satellite. Numerical operations require precise adherence to a coordinate system and specific force models.

Earth Orientation Parameters (EOP) data is a cornerstone for accomplishing the inertial to fixed transformation of coordinates. Most numerical integrators integrate in an inertial frame, while the force models are applied in a fixed frame. The EOP information consists of  $\Delta UT1$ , the difference between UT1 (Universal time) and UTC (Coordinated Universal time), the length of day, polar motion coefficients ( $x_p$ ,  $y_p$ ) describing the movement of the Earth's rotation axis to the crust, and  $\Delta AT$ , the number of leap seconds between UTC and TAI (Atomic time). There are typically small differences between sources of EOP data (e.g., International Earth Rotation Service - IERS, National Geospatial Intelligence Agency - NGA, and US Naval Observatory - USNO), but the impact on overall accuracy is usually small (a few meters or so).

Space weather data is the primary input, other than satellite characteristics, for atmospheric drag models. Even the smallest changes in space weather data can have large effects during propagation.

### OBJECTIVE

This paper investigates the available data, compares products within and between organizations, and provides recommendations on which data should be used for past, present, and future satellite operations. Specific instructions are provided to assemble the data into input data files and the location of completed files on the Internet is discussed.

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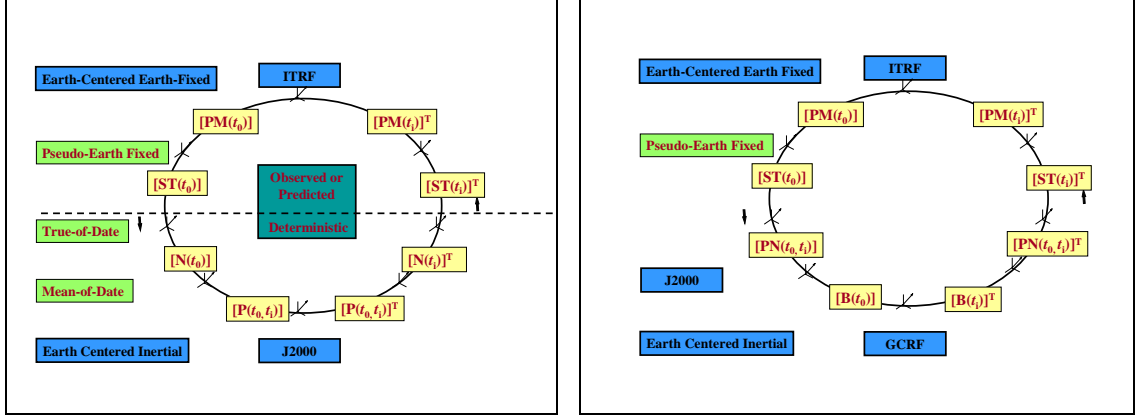
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## BACKGROUND

### Earth Orientation Parameters (EOP):

The EOP data assists in the transformation between inertial and fixed coordinate systems. For this paper, I'll simply use ECI and ECEF to denote the two systems, respectively. Figure 1 shows a notional representation of the FK5 and IAU2000 coordinate systems. EOP data is the foundation of these calculations.



**Figure 1. Coordinate System Representation.** These plots show the general schematic for transforming vectors in the J2000 and IAU2000 coordinate systems. The primary objective is the transformation between inertial and fixed coordinate systems. Polar motion, sidereal time (indirectly), and nutation all use EOP data for solution.

The specific equations are lengthy (Vallado 2004, 205-228), but in summary form, the EOP data enables every operation beginning with the initial determination of the various time systems. Recognize that the time arguments are found using EOP, and these in turn are used to determine the remaining coefficients of each transformation. For IAU 2000,

$$(yr, mon, day, UTC, \Delta UT1, \Delta AT) \Rightarrow (UT1, TAI, TT, T_{UT1}, T_{TT})$$

$$[PN] = \begin{bmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{bmatrix} ROT3(s)$$

$$[ST] = ROT3(-\theta_{ERA})$$

$$[PM] = ROT1(-s') ROT2(x_p) ROT1(y_p)$$

$$\bar{r}_{ITRF} = [PM]^T [ST]^T [PN]^T \bar{r}_{GCRF}$$

$$\bar{v}_{ITRF} = [PM]^T \left\{ [ST]^T [PN]^T \bar{v}_{GCRF} - \bar{\omega}_{\oplus} \times \bar{r}_{PEF} \right\}$$

### Space Weather Data:

The space weather data consists primarily of the geomagnetic indices ( $k_p$ ,  $a_p$ ), and the solar flux ( $F_{10.7}$ ). These indices provide atmospheric models with the necessary information to predict the atmospheric density for orbital calculations. Although the space weather data has a huge impact on the ultimate value of atmospheric density and there has been significant study over the years, the models are less than perfect. As Vallado (2005) showed, atmospheric propagation results can differ by 50-100 km simply by how the space weather parameters are treated within a program. The causes for these large discrepancies are varied, but include severely limited observational data with which researchers can investigate the phenomenon,

lack of good reference orbits, and lack of time and money to complete such studies. Vallado (2005) remarks that there is not a standard method of treating the data, but provides a proposed solution. Whatever the cause, it is appropriate to use the available information in the best way possible to maximize the chances of success for each use.

For atmospheric drag, the acceleration is simply one of many that are numerically integrated. The following equation shows this.

$$\bar{a}_F = -\frac{\mu \bar{r}}{r^3} + \bar{a}_{Non-spherical} + \bar{a}_{Drag} + \bar{a}_{3-body} + \bar{a}_{SRP} + \bar{a}_{Tides} + \bar{a}_{Albedo} + \bar{a}_{Other}$$

If we examine a particular density model, say the Jacchia 1970 model, the space weather data parameters ( $F_{10.7}, \bar{F}_{10.7}, k_p$ ) are used in several equations for exospheric temperature

$$T_c(K) = 379 + 3.24\bar{F}_{10.7} + 1.3(F_{10.7} - \bar{F}_{10.7})$$

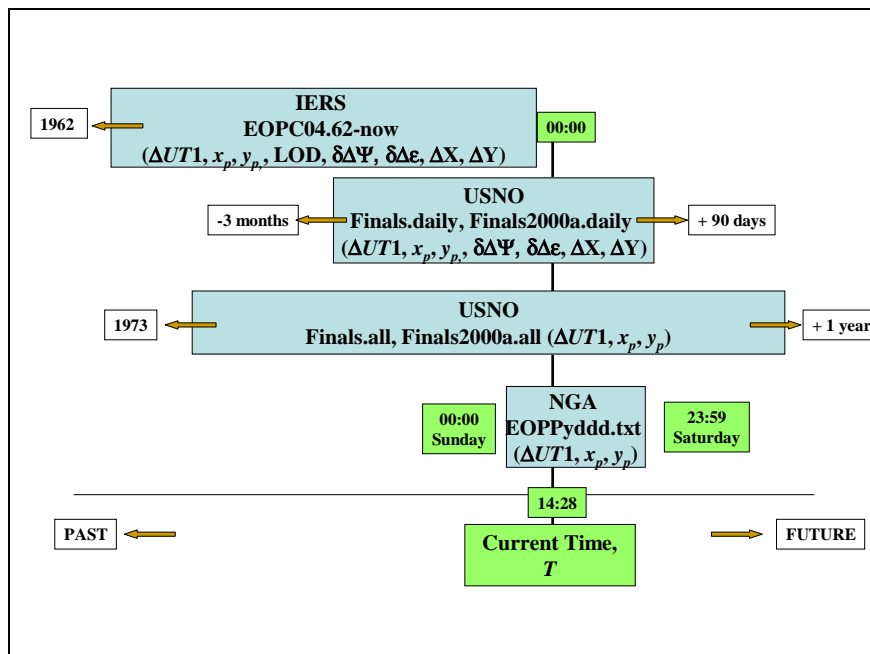
$$\Delta T_{corr} = 28.0^\circ k_p + 0.03 \exp^{k_p}$$

and density

$$\Delta \log_{10} \rho = 0.012 k_p + 1.2 \times 10^{-5} \exp^{k_p}$$

## EOP ANALYSIS AND DISCUSSION

The EOP data comes from three general sources: IERS, USNO, and NGA. Each covers a different time period, and some of the quantities are available only in certain data files. Figure 2 shows the time spans and available information. Note that satellite operations require seamless integration of this information.



**Figure 2. Earth Orientation Parameter Data Availability.** There are several sources of data we can use to populate a database of values for use in past, present, and future operations. File names are included from each source.

Each data provider gives slightly different information, although the core information is the same. For IERS, the data is (there is a corresponding file for the IAU2000 theory, containing corrections,  $\Delta X$  and  $\Delta Y$  instead of  $\delta\Delta\Psi$  (dPsi) and  $\delta\Delta\epsilon$  (dEpsilon)):

```

INTERNATIONAL EARTH ROTATION SERVICE
EARTH ROTATION PARAMETERS
EOP (IERS) C 04
FORMAT (2X, I4, 2X, A4, I3, 2X, I5, 2F9.6, F10.7, 2X, F10.7, 2X, 2F9.6)
*****
Date      MJD      x          y          UT1-UTC    LOD      dPsi      dEpsilon
          "          "          "          "          "          "          "
(0h UTC )
...
2004 JAN 1 53005 .031278 .153770 -.3895858 .0004983 -.054858 .000651
2004 JAN 2 53006 .028867 .153565 -.3900835 .0004346 -.055216 .000514
2004 JAN 3 53007 .026698 .153749 -.3904490 .0002619 -.055428 .000234
2004 JAN 4 53008 .024239 .154234 -.3906090 .0000719 -.055490 -.000096

```

The USNO also provides EOP data in the Bulletins A and B. Bulletin A contains the rapid results while Bulletin B contains the finalized results. There are two files, one for FK5, and one for IAU2000. It's significant to note that the USNO data is column specific, and it includes error estimates on the parameters, but no finalized value for the length of day. I've added column headings below (highlighted) to indicate which variables are listed. The file *finals.all* contains both the timing and polar motion values from January 2, 1973 to date, including about one year of predicted values.

#### Bulletin A from May 5

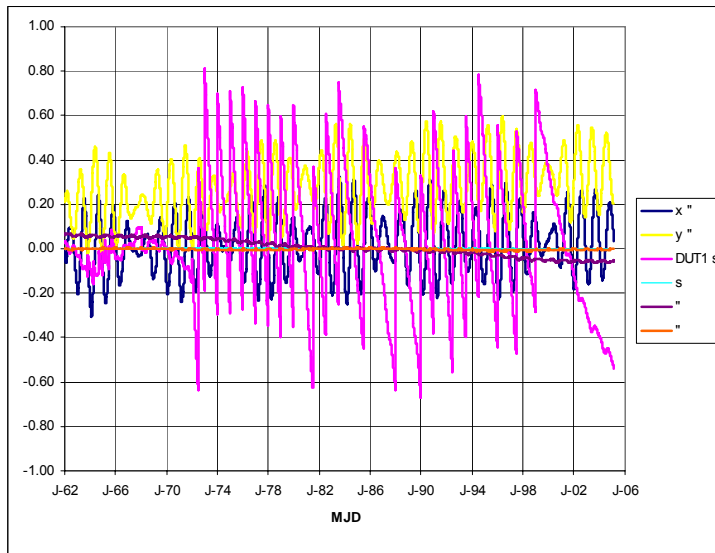
Date	MJD	xp	err	yp	err	DUT1	err	lod	err	dpsi	err	deps	err
4 1 1	53005.00	I .031244	.000048	.153837	.000038	I -.3896242	.0000036	0.4944	0.0025I	-56.182	.288	-.356	.340
4 1 2	53006.00	I .028844	.000055	.153666	.000035	I -.3900898	.0000036	0.4219	0.0037I	-56.267	.288	-.447	.340
4 1 3	53007.00	I .026642	.000053	.153845	.000027	I -.3904388	.0000065	0.2618	0.0029I	-56.230	.740	-.510	.340
4 1 4	53008.00	I .024196	.000061	.154296	.000027	I -.3906064	.0000045	0.0807	0.0040I	-56.014	.244	-.595	.340
4 1 5	53009.00	I .021488	.000060	.154846	.000020	I -.3906172	.0000046	-0.0493	0.0033I	-55.791	.244	-.810	.340
4 1 6	53010.00	I .018791	.000059	.155474	.000036	I -.3905345	.0000048	-0.0993	0.0034I	-55.682	.285	-1.079	.340

The Bulletin B values are in the same file as the Bulletin A values (at the end of each line). The finalized data lags about one to two months.

#### Bulletin B from May 5

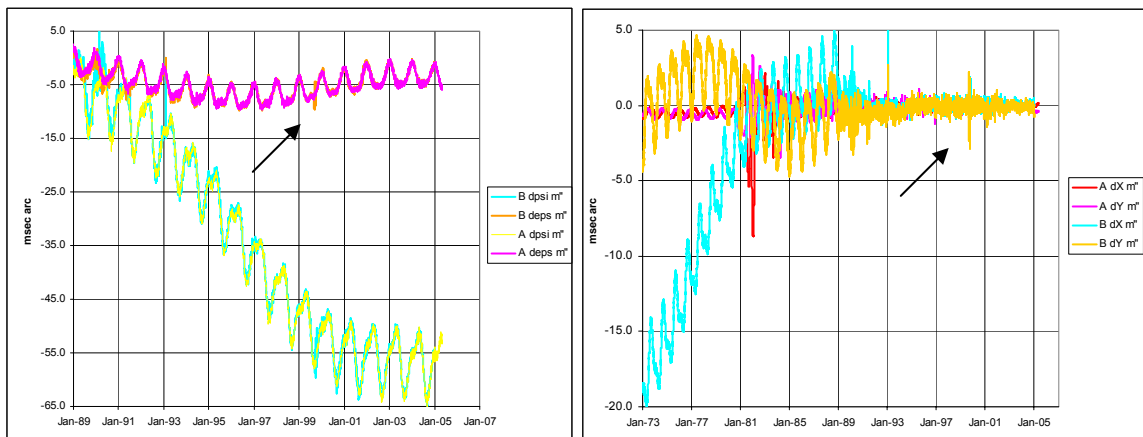
Date	Bulletin A	xp	yp	DUT1	dpsi	deps
...		0.031250	0.153770	-0.3895920	-56.100	-0.100
		0.028870	0.153550	-0.3900740	-56.100	-0.200
		0.026700	0.153740	-0.3904550	-56.000	-0.400
		0.024260	0.154230	-0.3906300	-55.900	-0.600
		0.021590	0.154800	-0.3906200	-55.700	-0.900
		0.018790	0.155440	-0.3905240	-55.500	-1.000

Before analyzing the data, it's useful to understand the behavior of the data over time. Figure 3 shows the various parameters for the last several decades.



**Figure 3. Historical EOP Values.** This figure shows the EOP parameters from 1962. Notice the centered motion about zero for  $x_p$ , the slight secular increase for  $y_p$ , and the continual downward trend for  $\Delta UT1$ . The large vertical spikes show the occurrences of leap seconds. The final 3 parameters (length of day and FK5 corrections) are very small.

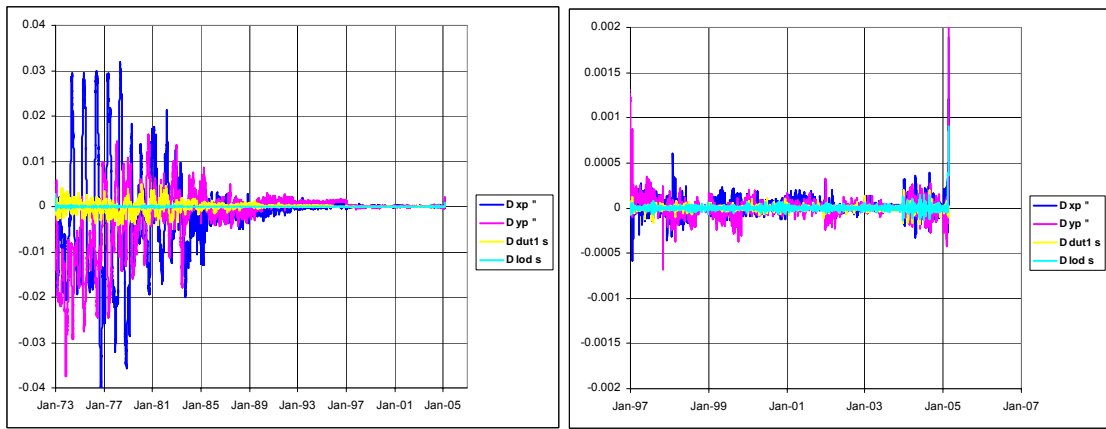
Comparisons between Bulletins A and B are possible as they contain similar parameters. The Bulletin A values “should” exhibit additional variations as they are the rapid calculations. However, this was not always supported by the data. Consider the following figures.



**Figure 4. USNO Comparisons between Bulletin A and Bulletin B Data.** Differences in the Bulletin A and B correction parameter values for the FK5 and IAU2000 theories are shown. Data is available for FK5 from Jan 2, 1973 and for IAU2000 from Jan 1, 1989. Older data shows larger differences as the current theories didn’t exist during those times. However, notice the unusual spike in the Bulletin B data for deps and dX. Both of these occur in about the September 1999 timeframe.

Although the differences show in a graph with a small scale, the relative difference in these parameters is remarkably small.

Next, we can examine similar parameters between the USNO and IERS (EOPC04) data.



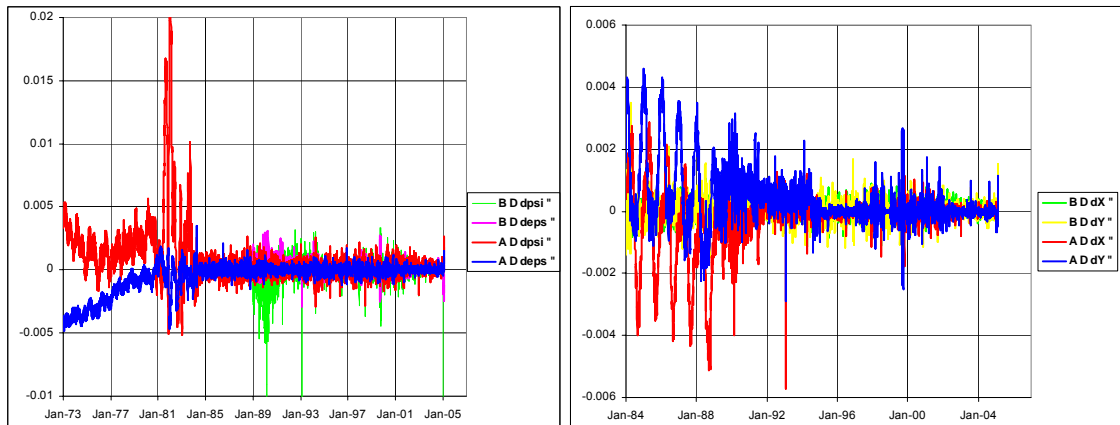
**Figure 5. IERS and USNO EOP Comparisons.** The IERS and USNO values for polar motion, delta time, and length of day are shown. Notice the similarity of the values after about 1997. The last few daily values from USNO exhibit additional variation over the comparable IERS values, as shown in the right-hand plot.

The data in Fig. 5 was taken in late February 2005. By May, the final values were available. For the last few days' values, Table 1 lists the results.

**Table 1. IERS and USNO Point Comparisons.** The difference between predicted and observed values is illustrated for February 22, 2005. Notice the “consistent” nature of the IERS results over time, while the USNO results appear to change more.

	IERS 22 Feb 05	IERS 5 May 05	IERS Delta	USNO 22 Feb 05	USNO 5 May 05	USNO Delta
$x_p$	0.035728	0.035699	0.000029	0.033830	0.035491	-0.031661
$y_p$	0.205453	0.205616	-0.000163	0.205647	0.205832	-0.000185
$\Delta UT1$	-0.5371265	-0.5371082	-0.0000183	-0.5370027	-0.5371187	0.000116

Comparing the corrections to FK5 and IAU2000 theories between IERS and USNO,



**Figure 6. IERS and USNO Correction Comparisons.** These parameters show little variation, although the Bulletin A values seem to exhibit slightly more variation, as would be expected.



The IERS provides uncertainties for each of the EOP parameters, shown in Table 2. These values are all within the differences seen in the previous figures.

**Table 2. Earth Orientation Parameter Data Accuracy.** This table lists the expected accuracy of the EOP parameters for several different time periods. The units are given with each quantity.

	1962-1967	1968-1971	1972-1979	1980-1983	1984-1995	1996- Date
$x_p''$	.030	.020	.015	.002	.0007	.0002
$y_p''$	.030	.020	.015	.002	.0007	.0002
$\Delta UT1$ s	.0020	.0015	.0010	.0004	.00004	.00002
LOD s	.0014	.0010	.0007	.00015	.00003	.00002
$\delta\Delta\Psi''$	.012	.009	.005	.003	.0006	.0003
$\delta\Delta\varepsilon''$	.002	.002	.002	.002	.0006	.0003

Some operational programs rely on the NGA coefficients. For the current week, predictions of  $x_p$ ,  $y_p$ , and  $\Delta UT1$  are available from NGA in the form of polynomials. The information is updated every Thursday. The pole positions are accurate to about 0.01" after a month, and  $\Delta UT1$  predictions are accurate to a few milliseconds after about 20 days. Although pole positions may be calculated many months into the future, the same operation for  $\Delta UT1$  is not recommended.

The NGA Earth Orientation Parameter Prediction (EOPP) files are ASCII files named eoppyddd.txt where ddd is the day of year for the following Sunday (e.g., EOPP5205.TXT is calendar day 205, 24 July, of 2005). This is a new format as of 2005 week 25, replacing the older eoppyww.txt where **y** was the last digit of the year and **ww** was the week (Sunday, which differed from the ISO definition of the week) of the year (e.g., EOPP402.TXT is week 02 of 2004). The predictions are calculated on Thursday of each week (Wednesday when a federal holiday falls on a Thursday) to go into effect on the following Sunday. Single daily values are given after the coefficients. NGA re-introduced inclusion of zonal tide corrections into the data beginning with week 2 of 2005. This may change the accuracy slightly.

```

53154.00 .056293 .000000 .079979 .034045 -.028833 -.113721365.25
435.00 .343292 .000000 .030247 .117765 .083031 .026326365.25435.00
53370.00 -.523010 -.000381 .000514 -.000831 -.022000 .006000
.000607 .000855 .012000 -.007000 27.5600 13.6600 365.2500 182.6250
32 5205 53575 53572
53575 -.013034 .417876 -.6043788
53576 -.011681 .418738 -.6046355
53577 -.010318 .419577 -.6049358
53578 -.008945 .420394 -.6051611

```

There is not much information given with the parameters, although the readme file (<http://earth-info.nga.mil/GandG/sathtml/eoppdoc.html>) explains each field. The information is presented below for convenience. There are several constants in the file – the annual period, AnnPer = 365.25, Chandler period, ChnPer = 435.0, semi-annual period, SAnnPer = 182.625. Using a FORTRAN format statement and the variable names, each line is

```

FORMAT( F10.2, 6(F10.6), F6.2 )      -> ta, A, B, C1, C2, D1, D2, P1
FORMAT( F6.2, 6(F10.6), 2(F6.2) )   -> P2, E, F, G1, G2, H1, H2, Q1, Q2
FORMAT( F10.2, 6(F10.6) )           -> tb, I, J, K1, K2, K3, K4
FORMAT( 4(F10.6), 4(F9.4) )         -> L1, L2, L3, L4, R1, R2, R3, R4
FORMAT( I4, I5, I6 )               -> dat, EOPPwk, teff

```

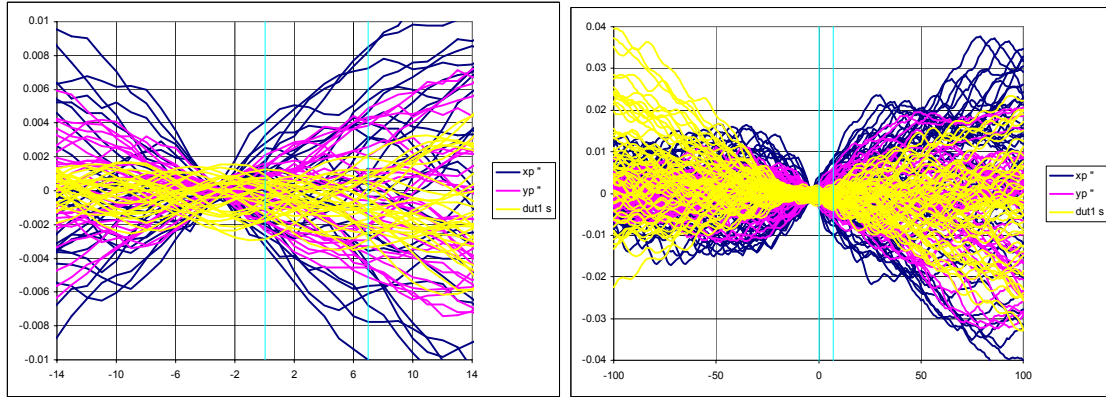
These coefficients are used with the following equations to obtain estimates of the EOP parameters at any time ( $t$  – the current time in MJD).

$$x_p(t) = A + B(t - ta) + \sum_{j=1}^2 C_j \text{SIN}\left(\frac{2p(t - ta)}{P_j}\right) + \sum_{j=1}^2 D_j \text{COS}\left(\frac{2p(t - ta)}{P_j}\right)$$

$$y_p(t) = E + F(t - ta) + \sum_{j=1}^2 G_j \text{SIN}\left(\frac{2p(t - ta)}{Q_j}\right) + \sum_{j=1}^2 H_j \text{COS}\left(\frac{2p(t - ta)}{Q_j}\right)$$

$$\Delta UT1(t) = I + J(t - tb) + \sum_{j=1}^4 K_j \text{SIN}\left(\frac{2p(t - tb)}{R_j}\right) + \sum_{j=1}^4 L_j \text{COS}\left(\frac{2p(t - tb)}{R_j}\right)$$

If we examine several runs of the data, we see the results in Fig. 7. Note in particular the lines for  $\Delta UT1$  in which the values increases over time. Extrapolating this for several months or a year is not recommended.



**Figure 7. Long-Term EOP Coefficient Performance.** Numerous weekly coefficients were used to calculate values that were then compared to actual EOP measurements. Notice that the week before the week of applicability shows the best match – a consequence of least squares processing.

Finally, leap seconds are included in the combined EOP file. Information is gathered from the Bulletin C that announces these periodic updates – <ftp://hpiers.obspm.fr/iers/bul/bulc/bulletinc.dat>. Be aware that the predicted EOP data doesn't immediately include this information.

### **Earth Orientation Parameters (EOP) Summary:**

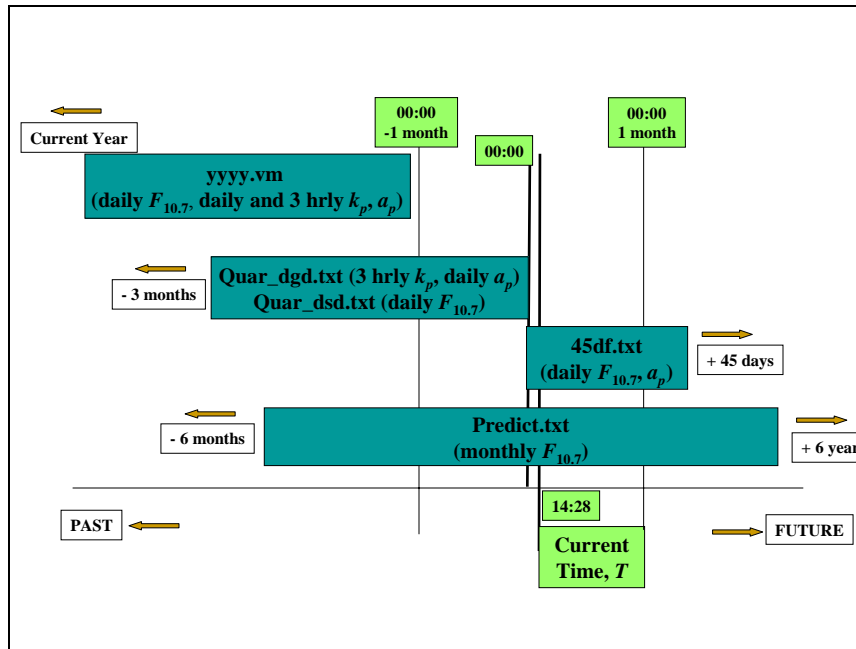
The most consistent set of final smoothed Earth Orientation Parameters ( $x_p$ ,  $y_p$ ,  $\Delta UT1$ , LOD) to within 1 day of the current time are available from IERS. The IERS Bulletins A (published weekly, includes one-year predictions for  $x_p$ ,  $y_p$ ,  $\Delta UT1$ ). The NGA data supports the current week only.

```
# Observed data
# 1. Current data - updated daily
# http://hpiers.obspm.fr/eoppc/eop/eopc04/eopc04.62-now
# http://hpiers.obspm.fr/eoppc/eop/eopc04/eopc04\_IAU2000.62-now
# Documentation can be found at:
# http://hpiers.obspm.fr/eoppc/eop/eopc04/EOPC04.GUIDE
#
# Predicted data (to +90 days) Bulletin A
# 2. Current data - updated daily
# http://maia.usno.navy.mil/ser7/finals.daily
# http://maia.usno.navy.mil/ser7/finals2000A.daily
#
# Predicted Data (to +1 year) Bulletin A
# 3. Current data - updated weekly
# http://maia.usno.navy.mil/ser7/finals.all
# http://maia.usno.navy.mil/ser7/finals2000A.all
#
# Documentation for Bulletin A can be found at:
```

# <http://maia.usno.navy.mil/ser7/readme.bulla>  
 # <http://maia.usno.navy.mil/ser7/readme.finals>  
 # <http://maia.usno.navy.mil/ser7/readme.finals2000A>  
 #  
 # 4. Current data - updated weekly (Thursday)  
 # <ftp://164.214.2.65/pub/gig/pedata/EOPPyyyy/EOPPyddd.TXT>  
 # Documentation can be found at:  
 # <http://earth-info.nima.mil/GandG/sathtml/eoppdoc.html>

## SPACE WEATHER DATA ANALYSIS AND DISCUSSION

Space weather data is available through the National Geophysical Data Center (NGDC) in the National Oceanic and Atmospheric Administration (NOAA), but there are several files necessary to piece together a complete data file. We can show a schematic for what data is available for propagation activities. Because there is 3-hourly data available for the geomagnetic indices, a truly current file requires frequent retrieval and organization of the data. Figure 8 shows the various files and their time of applicability.



**Figure 8. Space Weather Parameter Data Availability.** There are several sources of data we can use to populate a database of values for use in past and future propagations. Satellite operations require seamless integration of this information.

There are several things to consider when examining the atmospheric data. First, it's instructive to review the sensitivity of orbits to the incoming data (see Vallado 2005). It's also important to look at the trends in the data. This leads to a discussion of predicted values—a topic that always evokes varied responses. Finally, we need to determine how to “splice” the available data together. Vallado (2004, App D) discusses the various files needed for space weather data selection.

NOAA provides actual and predicted atmospheric parameters. The files for a particular year are designated as yyyy where yyyy is the year. Data within the current year are indicated with a version extension for the month, for instance 2004.v5 includes data through the end of May in 2004. The data are given to within about one month of the current time (data from Feb 22, 2005, and the header is added for clarity of variables). Note that the  $F_{10.7}$  values are valid at 1700 UT to May 31, 1991. From that time to the present, the values are for 2000 UT, measured at the Penticton site in Canada. All data is adjusted to 1.0 AU.

```

YrMoDy      kp          avg ap          sum          f10.7
0501212340183320232327678073347 18 7 9 9 12111207154 661.77 45109.90
0501222340195760372737334030320 67 80 22 12 22 18 27 15 331.36 31 99.10
0501232340203737332733402733267 22 22 18 12 18 27 12 18 191.05 26 92.80
0501242340213017172030302017180 15 6 6 7 15 15 7 6 100.52 28 91.70
05012523402210 31010 7101710 77 4 2 4 4 3 4 6 4 40.10 32 91.20
050126234023 7 3 3 7 7 3 7 7 43 3 2 2 3 3 2 3 3 30.00 23 86.60
050127234024 3 0 3 3 0 71013 40 2 0 2 2 0 3 4 5 20.00 20 84.30
0501282340252317 7 7 7 72727120 9 6 3 3 3 3 12 12 60.31 20 82.40
0501292340263320304033304043270 18 7 15 27 18 15 27 32 201.05 19 83.90
0501302340274043303330272020243 27 32 15 18 15 12 7 7 170.94 22 82.90
0501312341 133273037474030 7250 18 12 15 22 39 27 15 3 191.05 23 83.60

```

For the last month or so, solar and geomagnetic data files are required. The quarterly solar data provides only the daily  $F_{10.7}$  values.

```

:Product: Daily Solar Data          quar_DSD.txt
:Issued: 1425 UT 22 Feb 2005
#
# Prepared by the U.S. Dept. of Commerce, NOAA, Space Environment Center.
# Please send comments and suggestions to SEC.Webmaster@noaa.gov
#
# Quarterly Daily Solar Data
#
# Sunspot          Stanford GOES12
# Radio           Area          Solar X-Ray ----- Flares -----
# Flux           Sunspot       10E-6  New      Mean   Bkgd   X-Ray   Optical
# Date          10.7cm Number   Hemis. Regions Field Flux   C  M  X  S  1  2  3
#-----
2005 01 01    99      51      230      0    -999   B1.3   1  0  1  1  0  0  0
2005 01 02   100      52      250      0    -999   B1.9   2  0  0  0  0  0  0
2005 01 03    94      43      160      0    -999   B1.4   3  0  0  1  0  0  0
...
2005 02 16   113      61      720      0    -999   B1.4   2  0  0  1  1  0  0
2005 02 17   111      51      620      0    -999   B1.3   1  0  0  0  0  0  0
2005 02 18   104      46      590      0    -999   B2.1   5  0  0  1  0  0  0
2005 02 19    99      51      520      0    -999   B1.6   1  1  0  0  0  0  0
2005 02 20    96      60      580      1    -999   A7.0   0  0  0  0  0  0  0
2005 02 21    95      33      560      0    -999   A9.9   0  0  0  0  0  0  0
2005 02 22    -1      -1        0      -1    -999   *      0  0  0  0  0  0  0

```

The geomagnetic data provides the 3-hourly  $k_p$  and daily  $a_p$  values. Notice that the 3-hourly  $a_p$  values are not given.

```

:Product: Daily Geomagnetic Data    quar_DGD.txt
:Issued: 1830 UT 22 Feb 2005
#
# Prepared by the U.S. Dept. of Commerce, NOAA, Space Environment Center.
# Please send comment and suggestions to SEC.Webmaster@noaa.gov
#
# Current Quarter Daily Geomagnetic Data
#
# Middle Latitude          High Latitude          Estimated
# - Fredericksburg -     ---- College ----     --- Planetary ---
# Date          A      K-indices          A      K-indices          A      K-indices
2005 01 01    10  1 3 2 2 2 2 3 3    34  4 5 3 4 5 3 6 3    15  1 4 3 2 3 3 4 3
2005 01 02    20  3 3 3 3 2 4 3 5    64  2 3 7 6 5 7 4 5    33  4 4 5 4 3 5 3 5
2005 01 03    14  3 3 2 3 4 3 2 2    44  3 3 6 6 6 5 4 2    22  4 4 3 3 5 4 3 2
...
2005 01 31    15  3 2 3 4 4 3 2 1    43  2 2 3 6 7 6 3 1    19  3 2 3 4 5 4 3 1
2005 02 01     4  3 0 1 1 1 1 1 1     6  2 0 0 3 3 2 0 1     6  3 0 1 2 1 2 2 2
2005 02 02     7  1 2 2 2 1 2 3 1     8  0 0 3 3 2 3 2 1     8  1 1 3 2 2 2 3 1
2005 02 03     5  1 3 2 1 1 1 1 0     9  0 4 4 3 1 0 1 0     8  1 4 3 1 1 2 1 1
...
2005 02 20     6  3 2 2 0 1 1 2 2    16  4 3 3 1 3 4 3 2    12  4 3 3 0 2 3 2 2
2005 02 21     4  3 2 2 0 1 0 0 1    18  2 3 3 4 4 2 4 3     8  3 3 1 2 1 2 1 2
2005 02 22    -1 -1-1-1-1-1-1-1-1    -1 -1-1-1-1-1-1-1-1    -1  1 1 0 0 1 2 -1-1

```

Predicted values of solar activity are very useful. The predictions for the next 45 days are found in the 45df.txt file. Notice there are only single daily values, and no  $k_p$  values.

```

:Product: 45 Day AP Forecast 45DF.txt
:Issued: 2005 Feb 21 2059 UT
# Prepared by the U.S. Air Force.
# Retransmitted by the Dept. of Commerce, NOAA, Space Environment Center.
# Please send comments and suggestions to SEC.Webmaster@noaa.gov
#
#
#          45-Day AP and F10.7cm Flux Forecast
#-----
45-DAY AP FORECAST
22FEB05 010 23FEB05 012 24FEB05 012 25FEB05 012 26FEB05 012
27FEB05 012 28FEB05 015 01MAR05 020 02MAR05 008 03MAR05 008
04MAR05 008 05MAR05 008 06MAR05 012 07MAR05 025 08MAR05 015
09MAR05 012 10MAR05 010 11MAR05 010 12MAR05 008 13MAR05 010
14MAR05 008 15MAR05 008 16MAR05 012 17MAR05 015 18MAR05 012
19MAR05 010 20MAR05 005 21MAR05 005 22MAR05 008 23MAR05 015
24MAR05 025 25MAR05 020 26MAR05 020 27MAR05 020 28MAR05 008
29MAR05 008 30MAR05 008 31MAR05 008 01APR05 008 02APR05 012
03APR05 025 04APR05 015 05APR05 012 06APR05 010 07APR05 010
45-DAY F10.7 CM FLUX FORECAST
22FEB05 095 23FEB05 090 24FEB05 085 25FEB05 085 26FEB05 085
27FEB05 085 28FEB05 090 01MAR05 085 02MAR05 085 03MAR05 085
04MAR05 085 05MAR05 090 06MAR05 090 07MAR05 095 08MAR05 095
09MAR05 100 10MAR05 100 11MAR05 100 12MAR05 100 13MAR05 100
14MAR05 100 15MAR05 100 16MAR05 100 17MAR05 100 18MAR05 095
19MAR05 095 20MAR05 090 21MAR05 090 22MAR05 085 23MAR05 085
24MAR05 085 25MAR05 085 26MAR05 085 27MAR05 085 28MAR05 085
29MAR05 085 30MAR05 085 31MAR05 085 01APR05 085 02APR05 085
03APR05 090 04APR05 090 05APR05 095 06APR05 095 07APR05 095

```

Long-range predictions (about 6 years into the future) are found in the predict.txt file. Notice there are no long-range geomagnetic predictions, and the solar flux has only a single value for each month. Given the difficulty in accurately predicting the geomagnetic indices, this is to be expected.

```

:Predicted_Sunspot_Numbers_and_Radio_Flux: Predict.txt
:Created: 2005 Feb 01 1800 UTC
# Prepared by the U.S. Dept. of Commerce, NOAA, Space Environment Center (SEC).
# Please send comments and suggestions to sec.webmaster@noaa.gov
#
# Sunspot Number: S.I.D.C. Brussels International Sunspot Number.
# 10.7cm Radio Flux value: Penticton, B.C. Canada.
# Prediction values are based on ISES cycle 23 forecast of 13-month running
# smoothed values.
# Current interpolation prepared by IPS Radio and Space Services, Australia
#
# Missing or not applicable data: -1.0
#
#          Predicted Sunspot Number And Radio Flux Values
#          With Expected Ranges
#
#          -----Sunspot Number-----      ----10.7 cm Radio Flux----
#  YR MO   PREDICTED   HIGH   LOW   PREDICTED   HIGH   LOW
#-----
2004 08        38.9    39.9    37.9    104.2    105.2    103.2
2004 09        36.5    39.5    33.5    101.6    104.6    98.6
2004 10        34.3    39.3    29.3     99.1    104.1    94.1
...
2007 10        15.6    30.6     0.6     75.7     98.7    52.7
2007 11        18.3    33.3     3.3     77.4    100.4    54.4
2007 12        21.3    36.3     6.3     79.4    102.4    56.4

```

Finally, it's important to realize that the solar flux data may be either observed (at the true Sun-Earth distance), or adjusted to 1.0 AU. The raw observed data (and adjusted in DAILYPLT.ADJ) may be found at

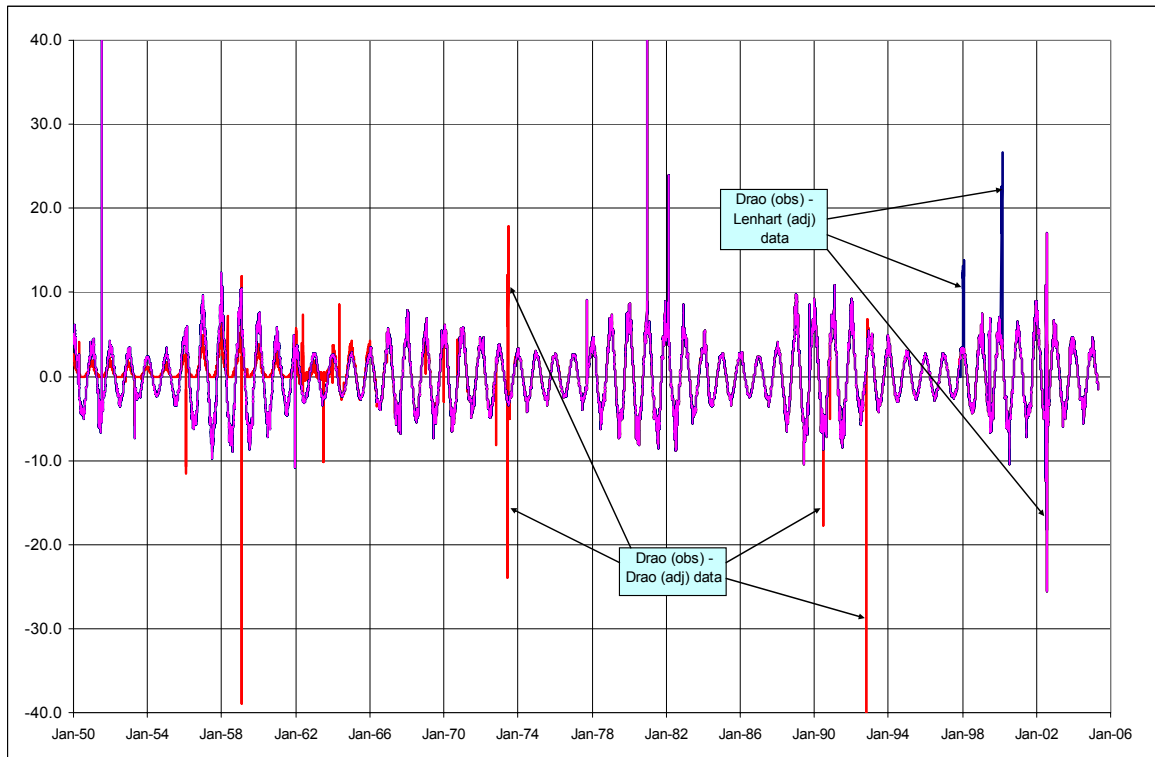
[ftp://ftp.ngdc.noaa.gov/STP/SOLAR\\_DATA/SOLAR\\_RADIO/FLUX/DAILYPLT.OBS](ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_RADIO/FLUX/DAILYPLT.OBS)

and the conversion uses the current Earth-Sun distance ( $r_{earth-sun}$ ) and the AU (149,597,870.66km).

$$F_{10.7}(obs) = \frac{F_{10.7}(adj)AU^2}{r_{earth-sun}^2}$$

Although this difference is small, some atmospheric models specifically require the solar flux at the true Earth-Sun distance. Therefore, our file includes both sets of data. The difference is large enough to require separate values for the 81-day average, and because many programs use either the centered or trailing 81-day averages, a total of six values are required for the solar flux.

Also note that the values found in the GEOMAGNETIC and the SOLAR FLUX directories on NGDC do not agree completely because they are actually different data. The GEOMAGNETIC files contain Lenhart adjusted data while the SOLAR FLUX files contain observed and adjusted Dominion Radio Astrophysical Observatory (DRAO) data (Knapp, 2005). The observed minus adjusted values of solar flux are shown in Fig. 9. The data spikes appear to be random in that no single data source appears to be “correct” for all times. Communication has been established with NGDC to better understand the differences and develop a common baseline for use.



**Figure 9. Difference of Observed and Adjusted Solar Flux Values.** The solar cycles are evident in the observed minus adjusted solar flux differences, along with the seasonal variations that drive the original variations over time. The data spikes occur in both observed and adjusted values, and in the DRAO and Lenhart data for a variety of reasons including multiple Sun-Earth distance corrections.

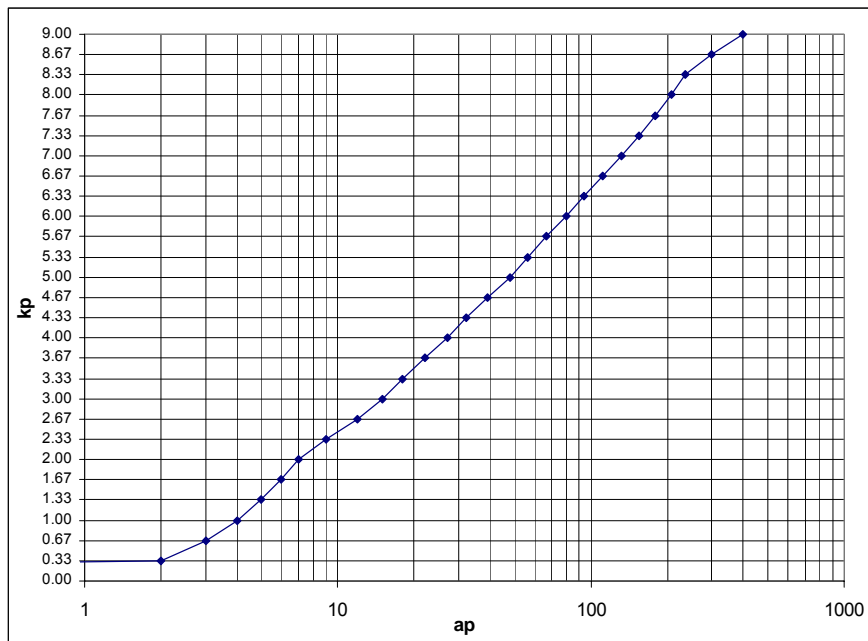
### Determining $k_p$ and $a_p$ values

Careful examination of the preceding data files reveals that there is not a single source that provides both  $k_p$  and  $a_p$  values for all periods. Thus, a practical means of converting between  $k_p$  and  $a_p$  is needed. Many papers have been written on this topic. Tanygin and Wright (2004) discuss the importance of interpolating these data with respect to filter applications and include an iterative approach based on a technical definition in the development of an atmospheric model (Jacchia, 1970). The process of finding  $k_p$  and  $a_p$  may seem easy at first, but on closer inspection, there are several nuances. Any approach should be able to convert in both directions—thus closure is important.

The primary problem arises from the need to determine values of  $k_p$  and  $a_p$  at a continuous number of time points, other than those actually measured. In over 50 years of recorded space weather data, there are only discrete recorded values of  $k_p$  and  $a_p$  quantities. Chapman and Bartels (1940) originally defined these discrete values. Note the plus and minus indicators represent 1/3 values and should not be rounded.

$k_p$	0o,	0+,	1-,	1o,	1+,	2-,	2o,	2+,	2-,	
$k_p$	0.0,	0.333,	0.667,	1.0,	1.333,	1.667,	2.0,	2.333,	2.667,	
$a_p$	0	2	3	4	5	6	7	9	12	
$k_p$	3o,	3+,	4-,	4o,	4+,	5-,	5o,	5+,	6-,	
$k_p$	3.0,	3.333,	3.667,	4.0,	4.333,	4.667,	5.0,	5.333,	5.667,	
$a_p$	15	18	22	27	32	39	48	56	67	
$k_p$	6o,	6+,	7-,	7o,	7+,	8-,	8o,	8+,	9-,	9o
$k_p$	6.0,	6.333,	6.667,	7.0,	7.333,	7.667,	8.0,	8.333,	8.667,	9.0
$a_p$	80	94	111	132	154	179	207	236	300	400

If one examines a plot of the quantities, it's readily apparent that the relation is not linear.



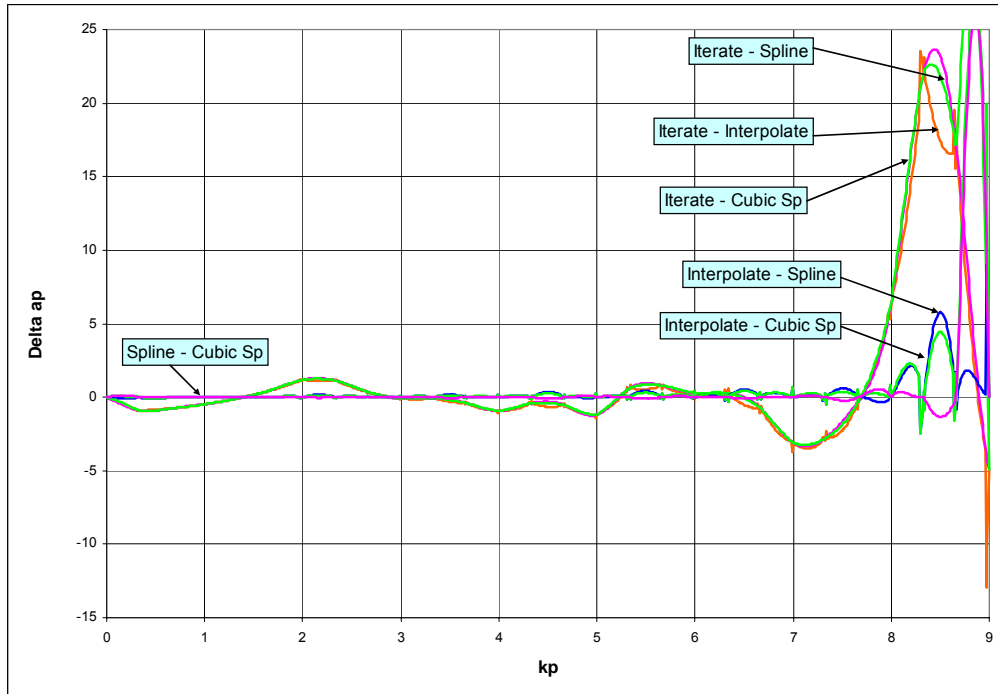
**Figure 10. Relationship between  $a_p$  and  $k_p$ .** The semi-logarithmic correlation between  $a_p$  and  $k_p$  is shown. Note that there are only distinct values that correspond between each scale and  $k_p$  is multiplied by 10.0.

Two practical problems arise. For the last month, only the 3-hourly  $k_p$  values are given, but any atmospheric model using  $a_p$  will need to convert the values, and when the values are finalized, they will be from the set of discrete values mentioned above. The second difficulty arises when assembling the predicted data, for which only  $a_p$  data is available. Specifically, values of  $a_p$  occur that do not have an exact corresponding  $k_p$  value per Chapman and Bartels (1940). To consistently use the data, a standard technique should be employed to convert the data to and from the  $a_p$  and  $k_p$  values, for both of these situations.

It could be argued that the indices have enough variability that to go to extreme lengths to preserve the original data is not fruitful. While I agree the data is seriously lacking in its rigor, it represents the best existing indices we have, and the one that all the models have been developed to over the years. As such, I think it would be unwise not to use the data as it's known today by introducing another variable into

the solution. However, if there is a demonstrated benefit of not using the existing defined values, NOAA should take the lead in changing the existing data to better support mission operations.

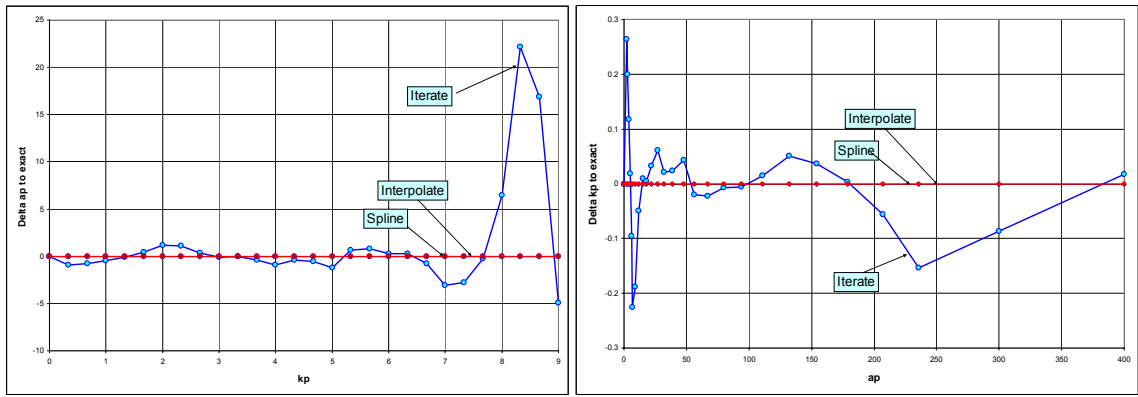
I explored several conversion approaches—a linear interpolation, an iterative approach, and two splining approaches (“Spline” by Press et al., 1992 and “Cubic” by Vallado 2004, 900). The iterative approach was introduced by Tanygin and Wright (2004) and it derives from Jacchia (1970). Comparing these approaches, there are large differences at the higher values of  $k_p$ , and many discrepancies throughout the interval. Vallado (2005) showed that numerical propagation relies heavily on the treatment of these data, so even small variations in the values could cause very different propagated results. Remember however, the indices themselves are not exact, and simply using them introduces uncertainty.



**Figure 11.  $a_p$  Difference Values between Conversion Techniques.** Differences in  $a_p$  results for several interpolation techniques are shown. Values from iteration, linear interpolation, and splining techniques are differenced.

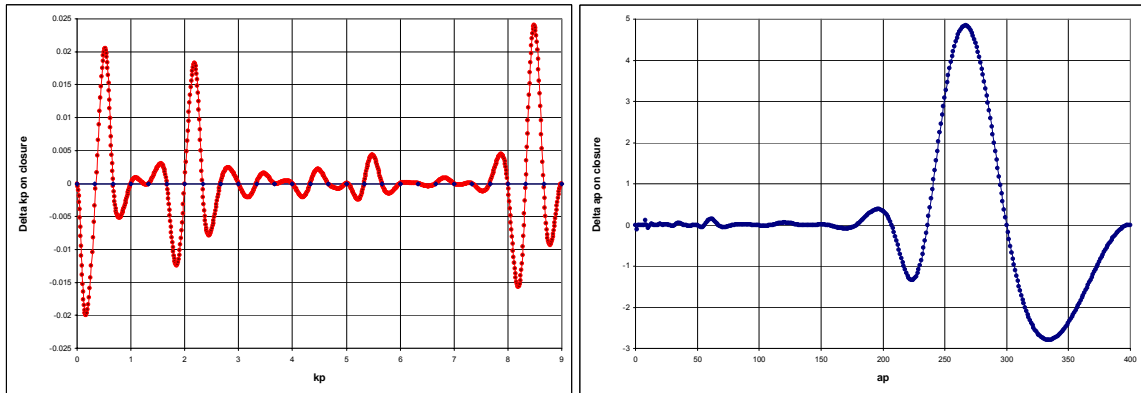
If we now examine the ability of each technique to replicate the original discrete values, we see that there are several large differences from the defined points. Note that the conversion is not always equal in both directions as the number of points for  $k_p$  is more evenly distributed than for  $a_p$ .





**Figure 12. Comparison of Conversion Techniques to Exact Values.** The exact values are the defined points of the scale by Chapman and Bartels (1940). Notice the discrepancy of the iterative approach, especially at the high  $k_p$  values (left). The  $a_p$  performance is a little better (right). All the other points lie on the zero horizontal axis.

The closure of iterative, interpolated, and cubic splining (Vallado, 2004, 900) values were exact, but the Numerical Recipes “Spline” technique showed some small variations. Note that the scale in Fig. 13 is significantly smaller than the previous graphs.



**Figure 13. Closure Properties for Numerical Recipes “Spline” Technique.**  $k_p$  and  $a_p$  closure properties are shown (left and right). There appears to be problems at some  $k_p$  and  $a_p$  values, but the scales are very small compared to differences from other interpolation techniques. The exact values all lie on the zero horizontal axis.

It appears that the cubic splining technique best replicates the observed data values, while simultaneously maintaining closure properties. The approach for the cubic spline (Vallado, 2004, 900) are shown below (note this is modified to match all four points instead of 2 points and slope).

Setup arrays of  $a_p = [0 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 12 \ \dots]$ ,  $k_p = [0 \ 0.33333 \ 0.66667 \ 1 \ 1.33333 \ 1.66667 \ \dots]$

Locate input  $k_p$  or  $a_p$  within the appropriate array given  $a_{pin}$  or  $k_{pin}$  (respectively)

Determine the 4 points for  $k_p$  and  $a_p$  (bracketing the input  $k_{pin}$  or  $a_{pin}$ )

$k_{p1}, k_{p2}, k_{p3}, k_{p4}, a_{p1}, a_{p2}, a_{p3}, a_{p4}$

Form coefficients for the cubic polynomials (one for  $a_p$  and one for  $k_p$ )

$k_0 = k_{p2}$   $a_0 = a_{p2}$   
 $k_1 = -k_{p1}/3.0 - 0.5*k_{p2} + k_{p3} - k_{p4}/6.0$   $a_1 = -a_{p1}/3.0 - 0.5*a_{p2} + a_{p3} - a_{p4}/6.0$   
 $k_2 = 0.5*k_{p1} - k_{p2} + 0.5*k_{p3}$   $a_2 = 0.5*a_{p1} - a_{p2} + 0.5*a_{p3}$   
 $k_3 = -k_{p1}/6.0 + 0.5*k_{p2} - 0.5*k_{p3} + k_{p4}/6.0$   $a_3 = -a_{p1}/6.0 + 0.5*a_{p2} - 0.5*a_{p3} + a_{p4}/6.0$

Solve the cubic polynomial for the real root ( $x$ ) between 0.0 and 1.0 for either  $k_p$  or  $a_p$  (both are shown for completeness)

$$0 = k_3x^3 + k_2x^2 + k_1x + k_0 - k_{pin} \qquad 0 = a_3x^3 + a_2x^2 + a_1x + a_0 - a_{pin}$$

Solve for either the  $a_p$  or  $k_p$  value (both are shown for completeness)

$$a_{pout} = a_3x^3 + a_2x^2 + a_1x + a_0 - k_{pin} \qquad k_{pout} = k_3x^3 + k_2x^2 + k_1x + k_0 - a_{pin}$$

### Long-Term Predictions

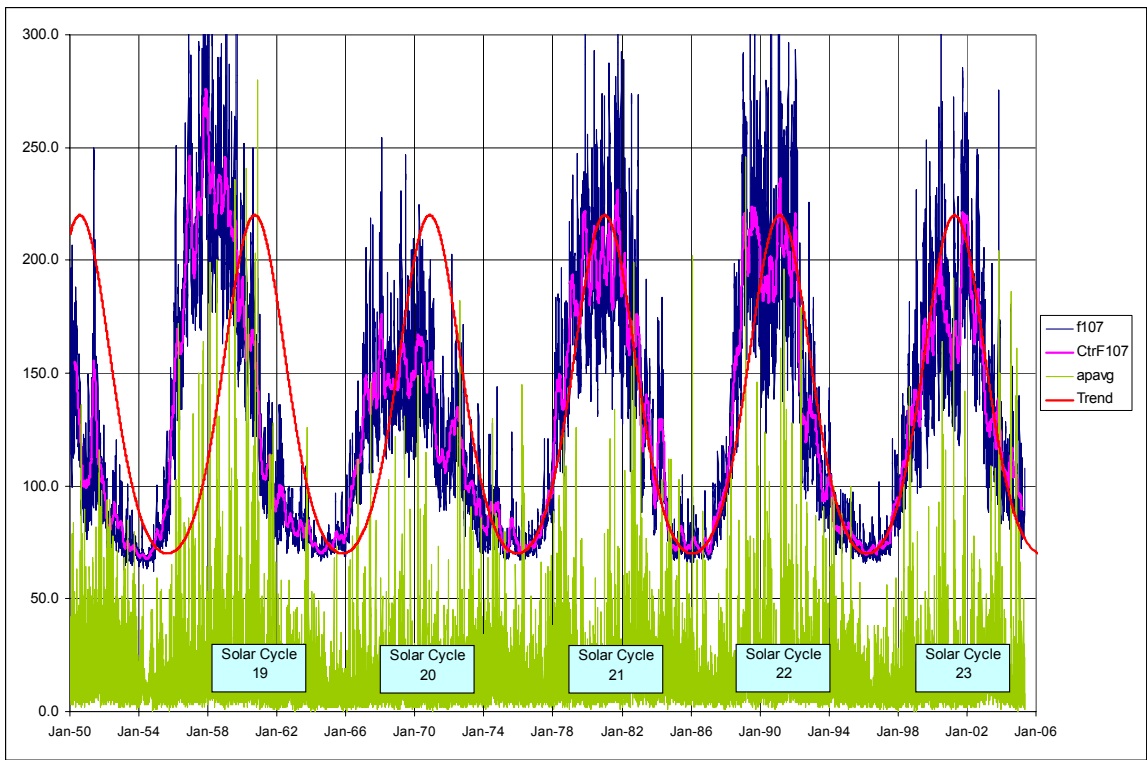
The long-term prediction of solar flux values is important for mission planning operations. However, the predictions are a complex analysis and combination of sunspot and geomagnetic activity, comparison to existing knowledge of solar activity since the 1700s, and several other factors (Hathaway et al., 1999). As such, the predictions have a great deal of uncertainty. Our goal was to find a simple, but “reasonably” accurate approach to determining long-term trends. We wanted to incorporate the existing prediction methods in the short-term, and use alternate approaches for the long-term (decades).

A commonly accepted set of predictions are the Schatten files. These files are available from the following website (password required) <ftp://fdf.gsfc.nasa.gov/generalProducts/database/>. The files generally span about one solar cycle and are periodically re-issued (about 3 to 4 times per year) to provide improved accuracy to the observed progress of each solar cycle. This is sufficient for many planning operations.

Vallado (2004: 535) introduced a polynomial trend that covers several solar cycles ( $t$  is the number of days from Jan 1, 1981). Note that the coefficients have been updated slightly from Vallado 2004 to better match the solar min region.

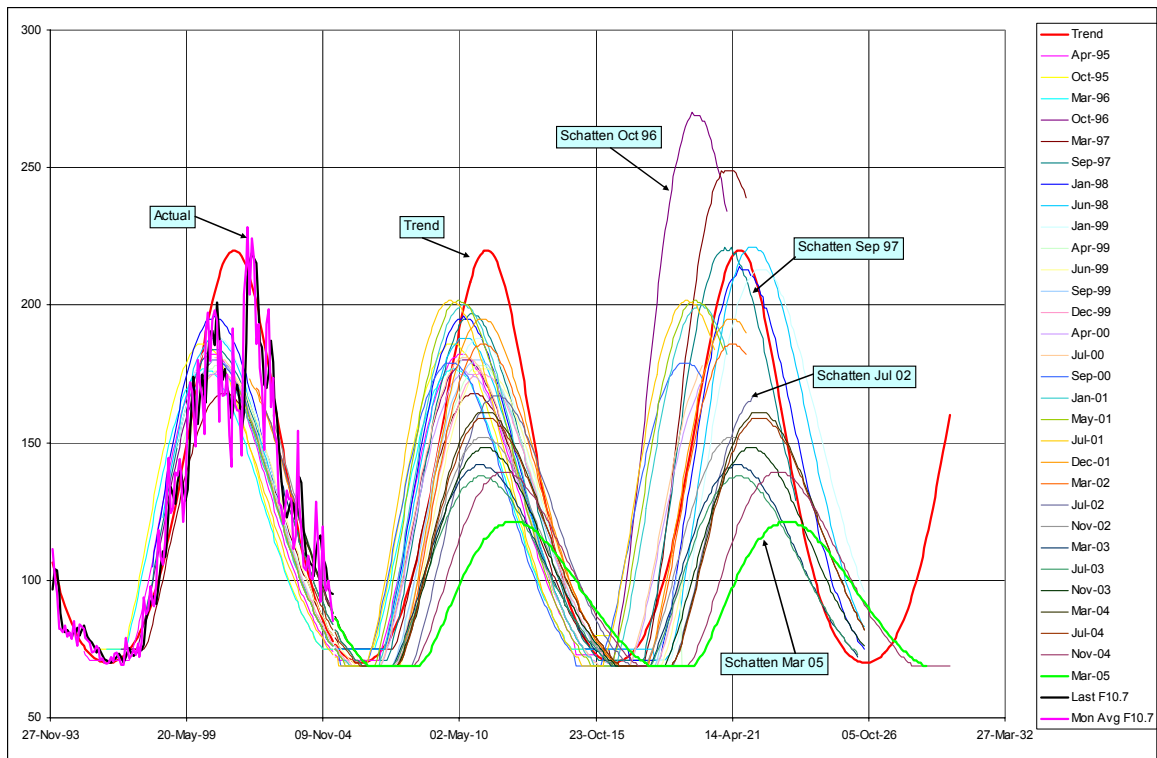
$$F_{10.7} = 145 + 75 * \cos\{0.001696 t + 0.35 * \sin(0.001696 t)\}$$

Figure 14 shows the solar flux and geomagnetic values over several solar cycles with the polynomial trend and centered 81-day average solar flux.



**Figure 14. Solar Flux Historical Values.** Solar cycles are shown from 1950 with the polynomial trend and centered 81-day average. The average  $a_p$  values are shown at the bottom. The solar cycle numbers begin in about 1750 when recorded sunspot data became routinely available.

Figure 15 shows the Schatten predictions with the actual and polynomial trend data. Because the Schatten predictions provide only monthly estimates of the solar flux, comparisons were made to the actual data over that same time.



**Figure 15. Recent Schatten Predictions.** Several Schatten predictions of solar flux are shown along with the polynomial trend. Notice the Schatten predictions cover one or two solar cycles.

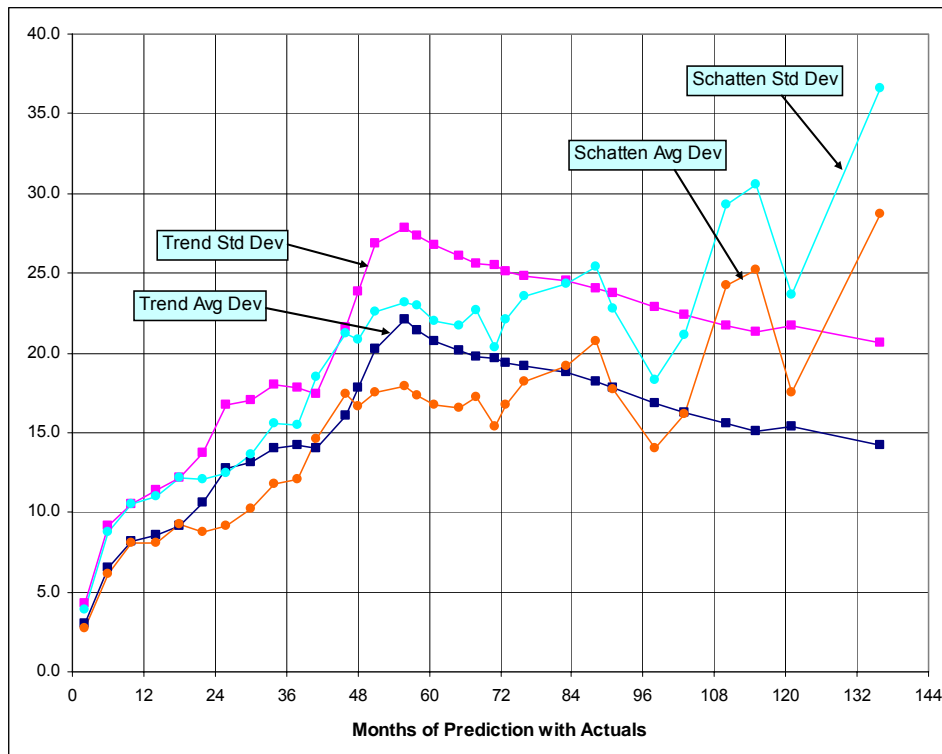
A study was conducted to determine the approximate performance of these predictions over the last three solar cycles.<sup>1</sup> In each case, the average deviation (average absolute difference from each value) and the standard deviation (of those differences) were used. 81-day and monthly averages were examined. Table 3 shows the results for about the last 3 solar cycles.

**Table 3. Estimated Space Weather Accuracy.** This table lists several quantities for the ‘accuracy’ of the space weather values. The 81-day average is compared to the actual daily values, as well as the trend, and monthly values. Statistics are formed over the last 3 solar cycles. The monthly trend is perhaps the most relevant number as it compares directly to the Schatten predictions.

	Avg Deviation	Standard Deviation
Ctr 81-day – Daily $F_{10.7}$	15.00	21.99
Last 81-day – Daily $F_{10.7}$	17.17	24.92
Trend 81-day – Daily $F_{10.7}$	20.79	29.88
Monthly Trend – Monthly $F_{10.7}$	15.33	21.85

The monthly trend information in Table 3 compares directly with the Schatten prediction values. Figure 16 shows the average and standard deviations for the trend and Schatten predictions.

<sup>1</sup> Note if Cycle 20 is included, the results would be different as the polynomial trend does not account for a varying solar cycle length. Hathaway et al., 1999, indicates that the solar cycle period can vary (by years) from the ‘common’ 11-year period. Currently, the estimates are uncertain until the solar cycle has about 3 years of observed data. It’s unclear if the most recent predictions can accurately model this variability.



**Figure 16. Schatten Prediction Performance.** The average and standard deviations are shown for several Schatten predictions, along with the polynomial (Trend) for the same monthly interval. All values use monthly averages of the  $F_{10.7}$  data. Notice as the number of actual data points in the Schatten predictions increases, the statistics become worse. However, the statistics are slightly better in the short term.

The above results were then compared with published information about the uncertainty of the predictions. From NOAA ([http://www.sec.noaa.gov/forecast\\_verification/F10.html](http://www.sec.noaa.gov/forecast_verification/F10.html)), the predicted one to three-day uncertainties are about 3-5 SFU at solar min, and 10-25 SFU near solar maximum (one-day, to three-day values). A few comparisons were also made with the 45df.txt and predict.txt files. Each exhibited similar variations to those shown in Table 3.

Given the variability and the frequency of updates for the NOAA and Schatten prediction files, coupled with the extreme sensitivity of propagation results using even very small differences in the space weather data file (Vallado 2005), it may be more convenient for some applications to use the simple polynomial trend when decade or longer predictions are required. As Table 3 and Fig. 16 show, the variability of the polynomial trend is about the same as the 81-day average data. In addition, the primary feature not modeled in the polynomial trend is the delay introduced by a longer than average solar cycle (e.g., Cycle 20). However if a longer cycle occurs for Cycle 24 or Cycle 25, the polynomial trend will predict an earlier occurrence for the solar maximum. Thus, any satellite designed on this prediction will have lower than anticipated fuel budgets, in the early portion of the mission, due to the lower atmospheric drag contribution.

A side note is the performance of the centered 81-day values versus the trailing 81-day average is warranted. It appears that the centered values 'slightly' better model the time-varying nature of the solar flux data. Many atmospheric models are designed and assume use of the centered 81-day values. However, operational centers often use the trailing 81-day average for practical reasons. Perhaps a mix, depending on the time span of interest, would be better.

## ***Space Weather Data Summary:***

To assemble a seamless file of space weather data, the following NOAA files should be used.

- Historical to within a month or two of the current time—General Directory:
  - [ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC\\_DATA/INDICES/KP\\_AP/](ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC_DATA/INDICES/KP_AP/)
  - Current data—updated Monthly (~22nd – 24th)
    - Each month is a yyyy.vx where x is the month (1-12) and yyyy is the year. When the year is complete, it's simply yyyy
    - [ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC\\_DATA/INDICES/KP\\_AP/2005.v1](ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC_DATA/INDICES/KP_AP/2005.v1)
    - Data assembled from 1950 to 2005 (atmosall.txt)
      - Numerous omissions exist in the data (above file is linearly interpolated)
  - Current data—updated daily
    - Includes observed F10.7 daily values
    - [ftp://ftp.ngdc.noaa.gov/STP/SOLAR\\_DATA/SOLAR\\_RADIO/FLUX/DAILYPLT.OBS](ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_RADIO/FLUX/DAILYPLT.OBS)
- Last month or two and initial predictions—General Directory:
  - <http://www.sel.noaa.gov/Data/index.html>
  - Current data—updated 3-Hourly
    - [http://www.sel.noaa.gov/ftplib/indices/quar\\_DGD.txt](http://www.sel.noaa.gov/ftplib/indices/quar_DGD.txt)
  - Current data—updated Daily (~0230 UTC)
    - [http://www.sel.noaa.gov/ftplib/indices/quar\\_DSD.txt](http://www.sel.noaa.gov/ftplib/indices/quar_DSD.txt)
  - Predicted data—updated Daily (~2114 UTC)
    - No 3hrly values
    - <http://www.sel.noaa.gov/ftplib/latest/45DF.txt>
- Long term predictions—General Directory:
  - Predicted data—updated Monthly (~3rd of the month)
    - <http://www.sel.noaa.gov/Data/index.html>
    - Includes F10.7 monthly values predicted for about 2 years into the future
    - Also has some old data which [depending on the access time] is overcome by actual measurements
    - <http://www.sel.noaa.gov/ftplib/weekly/Predict.txt>
  - Polynomial trend
    - Applicable for decades
    - May have a small jump between the 45df.txt prediction, but it will be within the uncertainty of either method

## **AVAILABILITY**

The process of assembling these files includes several steps. First, the files need to be downloaded. The new files can then be assembled. Once accomplished, this entire operation could be automated to provide the available Internet resources. Dr. Kelso provided the automation for this task and the results are located at

<http://celestrak.com/SpaceData/>

The space weather data is updated on CelesTrak every three hours at 35 minutes past the hour (e.g., 0035 UTC, 0335 UTC) since the 3-hourly data seems to come out at 30 minutes past the hour. A difficult task was determining the data file update frequency, and the reliability of data being available at the posted times. Most sites were pretty regular, but some are very erratic. The sites listed above will provide nearly real-time updates for both sets of files. The formats are given at the top of each file and will be consistent for use with the next release of Analytical Graphics Inc. Satellite Tool Kit (STK). Since the files are simple ASCII data, they may also be used with other applications.

## **CONCLUSIONS**

This paper has investigated the available data sources for EOP and space weather data. There are numerous sources of data for each, and arguments are presented for forming seamless data files for use in operations. The IERS data is recommended for historical EOP values, reserving the Bulletin A and B for predicted

values. Although the NGA coefficients are useful and included in the datasets, they do not appear to be accurate enough to support operations. A recommendation is made to use a splining technique to convert between  $a_p$  and  $k_p$  values. The ability to replicate the discrete values and the closure properties are cited as supporting rationale for this choice. A polynomial trend is presented and recommended for the formation of long-term solar flux values. Several statistics show it is very similar to the Schatten predictions that are published intermittently. The agreement over the last several solar cycles is very good, suggesting this could be an alternative for users wishing long term space weather predictions. The process has been automated by Dr. Kelso of AGI's CSSI group and the data files for both are available at <http://celestrak.com/SpaceData/>.

## ACKNOWLEDGMENTS

I would like to thank T.S. Kelso for automating this process and providing the Internet availability for the data. His work and dedication to foresee specific implementation issues and data availability permit robust and timely formation of these data files.

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