Part I: Shining a Light on Visual Magnitude

Patrick North, Novarah Kazmi

April 29th is International Astronomy Day, and we'll be outside, hoping for a clear night to see a sky full of stars. These brilliant sources of light have been observed for centuries. But how bright are these stars? If you wanted to know before you would have to take a catalog, a camera, and go out on a perfect night to capture the perfect picture. These days we don't have to go outside to make our measurements.

Systems Tool Kit (STK) with the Electro-Optical Infrared (EOIR) plugin tool can model the behaviors of sensors and measure various characteristics of stars. STK and EOIR have a plethora of capabilities and we will cover a sliver of them for this analysis, specifically features related to measuring the visual magnitude of stars.

We need to first understand visual magnitude (v_{Mag}). When a telescope or sensor looks up at the sky, the brightness of the objects it's looking at is called apparent magnitude. If we're only considering what the human eye can see, then we're measuring visual magnitude. This is a relative measurement on a log scale. It's a ratio of the irradiance (which is the energy of the source over a given area) of one object to another reference object. For stars, imagine the far away bright burning object viewed as a tiny pinprick in the sky. This means when we're measuring the visual magnitude we're actually measuring the signal from a source object and because not every sensor responds the same, the atmosphere is always in the way, we need to establish a baseline for our measurements.

We establish a baseline by measuring a known object. We need to know the visual magnitude of the object we're measuring, that way we can check to see how well our detectors are measuring a known signal. The most common star that is used for reference is Vega, but we can also use another known bright source in our field of view, or multiple known (less bright) sources in the field of view. The method can change, but we will use the same formula and calibration steps to complete our analysis.

How to get started:

Let's bring in our telescopes and lawn chairs and take a quantitative approach to talking about bright stars. Star brightness isn't the technical term, we mean visual magnitude (v_{Mag}) and irradiance (E) of the star. We will use v_{Mag} and E as the visual magnitude and irradiance of the object of interest and v_{Ref} and E_{Ref} as the visual magnitude and irradiance of a reference object respectively.

$$\text{Equation (1): } v_{Mag} = v_{Ref} - 2.5 * log_{10}(E/E_{Ref})$$

Alternatively, we can measure the signal from the image itself. Using this technique we'll look at the raw data from the EOIR synthetic scene and we'll use the measured signal (s_{Ref}) of our

object of interest in our formula. The conversation between the irradiance is discussed in the full report of this analysis in Part II.

$$\text{Equation (2): } v_{Mag} = v_{Ref} - 2.5 * log_{10}(s/s_{Ref})$$

Regardless of which equation we use, we expect our results to be the same. But we're jumping ahead of ourselves. Let's take a look at our STK scenario.

We mentioned that there are three approaches we can use:

1. Measure Vega
2. Measure another bright source
3. Measure multiple sources

It doesn't matter which method we choose, we first need to calibrate to our reference star or stars. We'll use the v_{Ref} value we find in our textbooks (Vega has a $v_{Ref} = 0.03$) and use EOIR to measure the E_{Ref} which we'll read off of our details panel. We can also export the raw data and get the s_{Ref} value.

From this simulated image we can see a single star shining brightly in the center of the synthetic scene. Because we're using a simulator we can look at the details and see what the in-band entrance aperture irradiance value is for our star:

We can read off the Inband Irradiance, for this sensor it is 1.140129e-12 W/cm$^{\wedge 2}$, but we have to do a little more leg work for the Raw Sensor Data. We'll export the raw sensor data file into excel and pull out the sum of the source peak from our file. Read the full process in Part II.

Once we have our reference value, all we need to do is fill in the equation with our calibrated terms and measure more stars. Wanna try it yourself?

Part II: Shining a Light on Visual Magnitude

Patrick North, Novarah Kazmi

Who doesn't like staring up at the stars? One of the most humbling experiences is looking up at the night sky and staring at the stars. Constellations of stars help us find out how we're oriented in the universe and we can use their relative positions and brightness to determine which star is which. In this write-up we are going to talk specifically about measuring the brightness of stars and other celestial objects using a metric called visual magnitude.

This write-up uses Systems Tool Kit (STK) with the Electro-Optical Infrared (EOIR) plugin to model the sensors and stars. STK and EOIR have a plethora of capabilities, some of which allow us to measure the irradiance reaching our sensor or simulate the digital output from a sensor model. Both the irradiance and digital signal let us calculate visual magnitude.
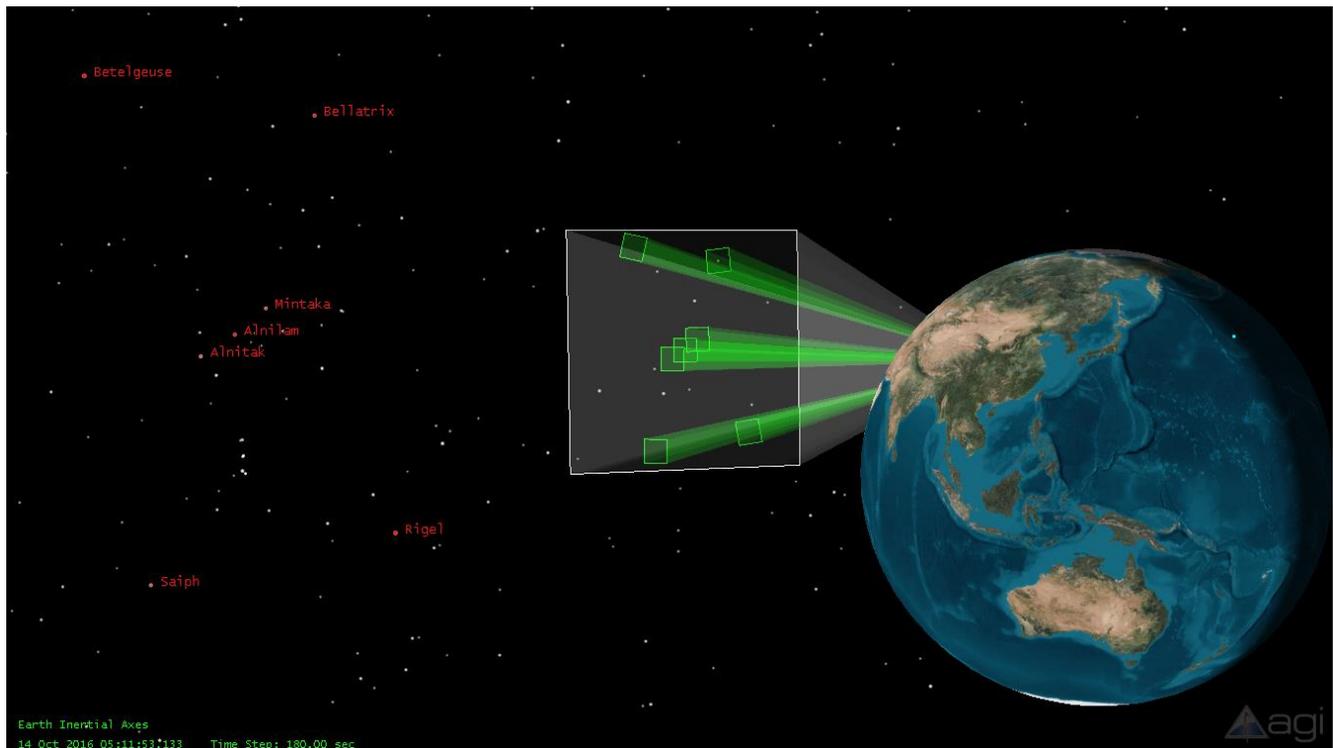
Figure 1 - A 3D STK view of the Orion constellation and the sensors on Earth pointing to it.

Abstract. In this article we cover a few different aspects of measuring and using visual magnitude as a metric from both an astronomical and imaging science perspective. This will include the basic calculation, calibrating a sensor or image to visual magnitude using reference stars, estimating an unknown object's visual magnitude, some of the complications involved, and common variations of the metric.

Keywords: Visual Magnitude, Vmag, Image Processing, EOIR, Sensor, Star, SSA.

What is Visual Magnitude: A way to measure brightness

Let's start with a few basic definitions. Apparent magnitude is the relative irradiance to some reference irradiance measured on a log scale. When the irradiance is integrated over the human visual response or v-curve, then this becomes visual magnitude (v_{Mag}). Visual magnitude or vmag can be thought of as the relative brightness of an object, measured on a log scale. Here brightness is defined for the human visual system using something called the human visual response curve, which weights the average human eye's response as a function of light color or wavelength. Characterizing the visual magnitude of distant

stars is relatively straightforward because they're glowing balls of energy emitting a broad spectrum of light in all directions equally, so we can treat them as isotropic point source emitters. Their irradiance or brightness is a combination of the star's temperature, size, and distance from the observer. We can measure the visual magnitude of other objects in the sky (such as planets, galaxies, or even man-made satellites), but the measurement is more complex because the brightness is a function of reflections involving material properties, shape, and relative distances and orientations of objects that cannot be observed directly. Once we have our vmag measurements we can use them to directly compare one measured brightness to another.
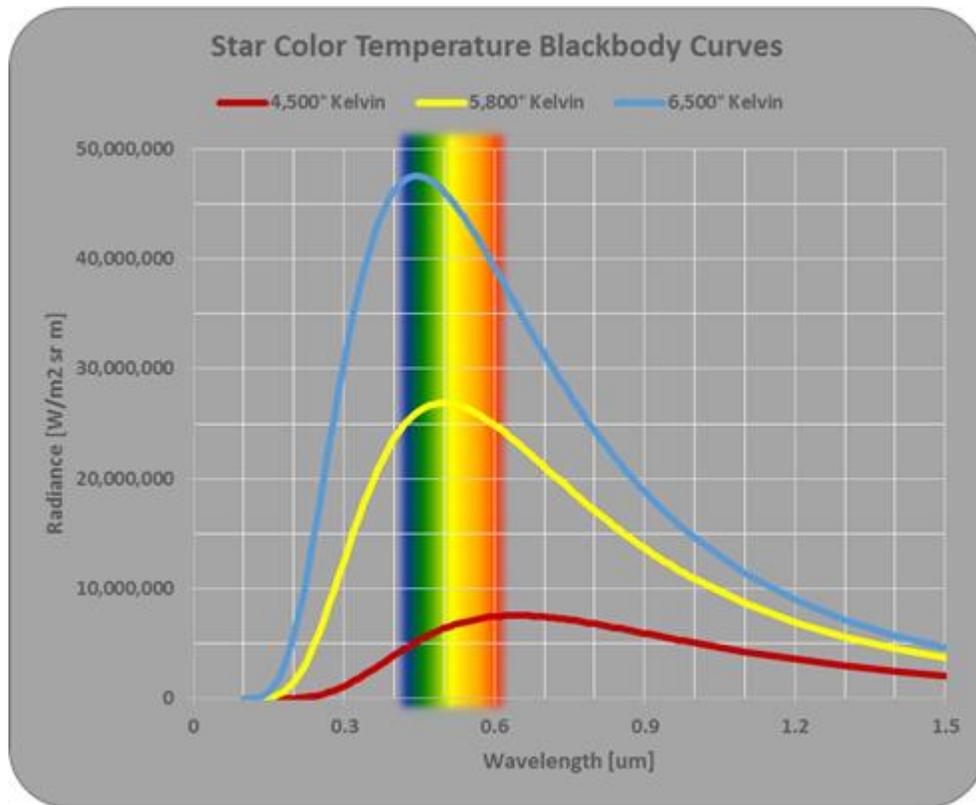
Figure 2 - Blackbody curves for stars at 3 peak temperatures.

Systems Tool Kit (STK) is our modeling and simulation tool that can model facilities, vehicles, aircraft, and satellites as well as different planets and stars. The STK plug-in Electro-Optical Infrared (EOIR) allows us to radiometrically model and simulate the emission of the stars and the detection characteristics of sensors we will use in this analysis. Within STK we can load specific stars and know exactly what their visual magnitude values are. By looking at these stars in the scenario with a sensor, we can generate a synthetic image to make visual comparisons about their relative brightness. It's qualitatively easy to sort stars by brightness and guess at which stars are about twice as bright or half as bright as others. We will show both real and simulated of images of well-known constellations throughout this analysis with labeled visual magnitudes to demonstrate this.
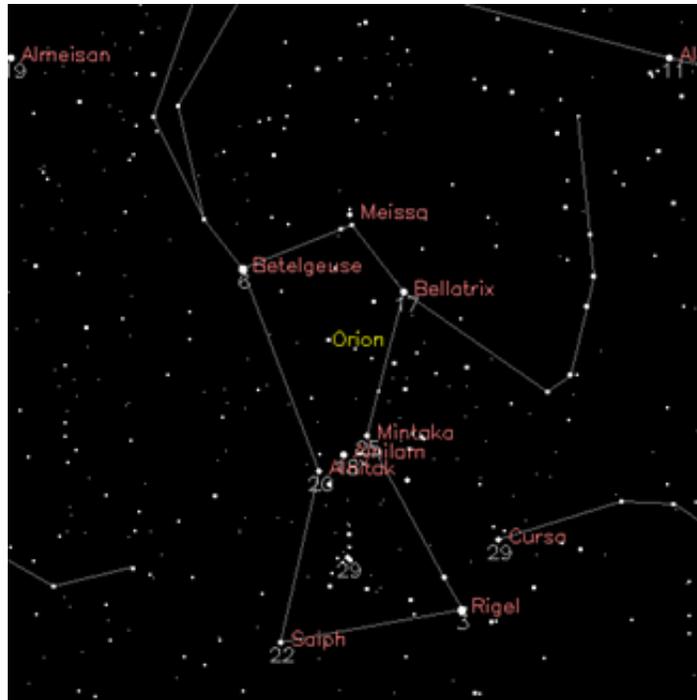
Figure 3 - Let's take a closer look at the Orion constellation. Orion constellation and neighboring stars with their visual magnitudes labeled. Image source Copyright © 1989 by Jef Poskanzer.

Qualitative analysis is fine, but what's more interesting is to use STK with EOIR to quantitatively measure the brightness of these stars. Using metrics such as irradiance and detected signal we can estimate the visual magnitude of these stars, which will be the focus of this analysis. We'll go into how this value is defined and calculated using the tools we have already mentioned.
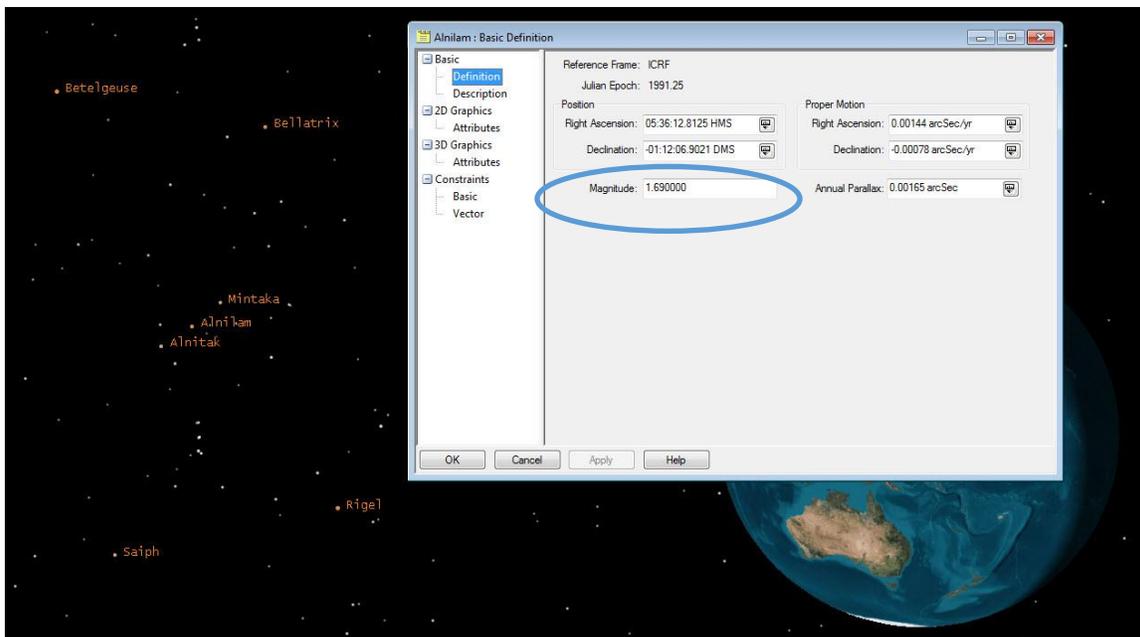
Figure 4 - STK image, analyzing the magnitude of stars in the field of view.

The Basic Equation: *Adding a little formalism*

When astronomers first started cataloging stars they selected the origin of the vmag scale (v_{Mag} = 0) as one of the brightest stars in the sky, Vega (these days Vega is given a catalog visual magnitude value of about 0.03). Dimmer stars were assigned positive numbers and brighter stars were assigned negative numbers. This method has been adjusted over time but has remained the standard for measuring visual magnitude. This plays out in our equation for v_{Mag}.

There are a few forms of the visual magnitude equation. We mentioned brightness before but in physics terms we'll be using measurements of the visible irradiance, the power per unit area in the visible spectrum range. Going forward, we will use v_{Mag} and E as the visual magnitude and irradiance of the object of interest and v_{Ref} and E_{Ref} as the visual magnitude and irradiance of a reference object respectively.

$$(1)\ v_{Mag} - v_{Ref} = -2.5\ \log_{10} \frac{E}{E_{Ref}}$$

To simplify this representation, the reference is often selected to be the zero visual magnitude point and that simplifies the definition to the following, with E_0 being the irradiance of a zero visual magnitude object.

$$(2)\ v_{Mag} = -2.5\ \log_{10} \frac{E}{E_0}$$

When taking pictures of the stars the imaging or camera system converts the irradiance of the object of interest into an image projected onto the focal plane. An image of a point source such as a star will have a total signal s that can be measured by detectors and converted into digital counts. The signal will be proportional to the irradiance reaching our camera (called the entrance aperture value). If we represent the conversion factor of Counts-to-Entrance-Aperture-Irradiance as C2E then we can rewrite the above vmag equations in terms of signal as:

$$(3)\ s = E * C2E$$

$$(4)\ v_{Mag} = -2.5\ \log_{10} \frac{E}{E_0} = -2.5\ \log_{10} \frac{s/C2E}{s_0/C2E} = -2.5\ \log_{10} \frac{s}{s_0}$$

Rather than having a perfectly calibrated camera and knowing the C2E value we can also use the signal of an object with known brightness to calibrate our system. When we use a non-zero visual magnitude, we need to reintroduce the v_{Ref} term. We can place it back into our equation and rearrange as in the following equation:

$$(5)\ v_{Mag} = -2.5\ \log_{10} \frac{S}{S_{Ref}} + v_{Ref}$$

When we calculate v_{Mag} we will use our reference star to define v_{Ref} and s_{Ref} . Some very bright and familiar common stars are listed in the table below. We will use their v_{Ref} values in our calculations and using the EOIR tool we can measure their s_{Ref} value for a given sensor and viewing conditions.

Calibration Star Target (Proper Name)	v_{Mag} (m$_v$)	Distance (ly)
Sirius	-1.44	8.60
Vega	+0.03	25.05
Rigel	+0.18	862.96
Betelgeuse	+ 0.45	641.8
Polaris	+1.97	432.63

Table 1 - Common stellar objects with accepted v_{Mag} and distance in light-years. Distance from earth is only one factor in determining Vmag. Star v_{Mag} values and distances are from the HYG Database.

Vmag Calibration Methods: *Finding the right reference*

Here we will present three methods to calibrate our images to estimate the visual magnitude. Each has pros and cons, and depending on the circumstances one may be more applicable than another.
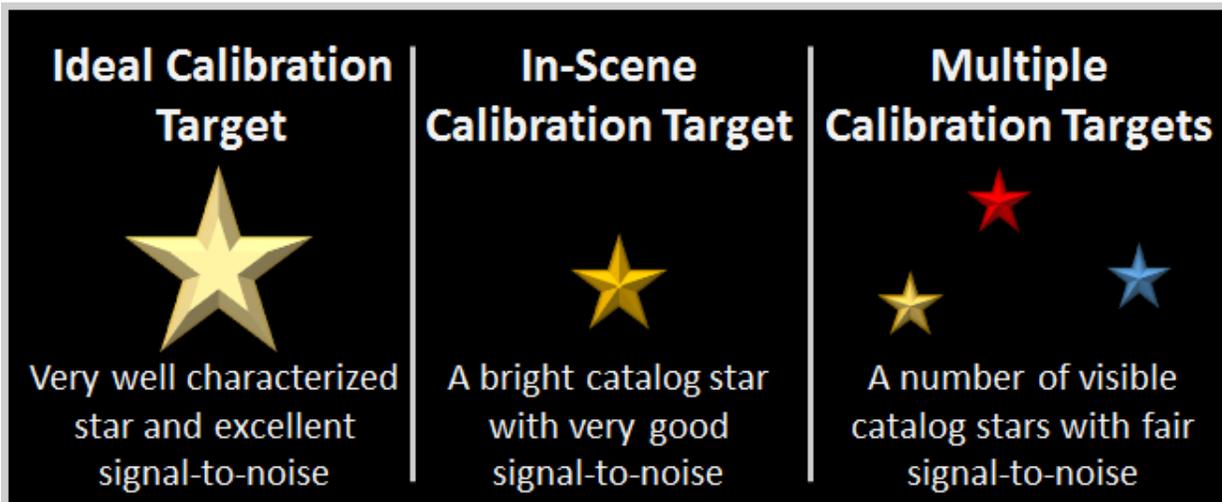
Figure 5 - The three tracks that will be discussed in this analysis.

The first method uses a very bright and well characterized star outside our object of interest's field-of-view. The pros would be it should be readily recognizable and provide an exception signal-to-noise ratio for any measurements. The cons with using one of these stars would be that they may not be anywhere

in the vicinity of your object of interest, and by changing the pointing of your system you will have a different atmospheric path for ground based sensors, also some time will pass and potentially change your system characteristics, and the reference and object signal may be so drastically different that the imaging parameters may need to be changed potentially invalidating the calibration.

The second method uses a potentially less bright but well characterized star in the vicinity of the object of interest, perhaps even within the same image or very close by. The pros would be that the reference star is still bright and should have good signal-to-noise and is in such proximity that the atmospheric path should be very similar, and also the time to steer the pointing from the reference to the target object should be very small if the reference and object of interest are not in the very same image. The cons are that the reference star may not be as well characterized, the atmospheric path may be slightly different, and the reference object signal may be a different magnitude than the desired target object.

The third method we'll talk about involves using multiple stars in the immediate proximity of the object of interest, ideally the same image. The pros would be very similar atmospheric paths for the object of interest and the reference stars and a range of available signals ideally similar to the target of interest. The cons would be that the references are somewhat arbitrary and may be poorly or even incorrectly characterized.

Method 1 - Calibrating a Sensor: *Working from a well-known star*

Vega is a simple and straightforward calibration star we can use for our v_{Mag} calculations. We could use other stars as our reference and calibrate the needed E_{Ref} and s_{Ref} terms however to keep things simple at first we will start with a star close to the origin and discuss other bright stars in the next section of this report.

Using a synthetic image generator gives us a quick way to perform a calibration to a known star by taking a picture of Vega. Doing this with STK using EOIR is shown below, where the 3D window on the left shows the general geometry of the sensor on the earth pointing towards Vega, the center window shows the position of the sensor at Place1 on the ground, and the right window shows the synthetic scene created by our simulated sensor.
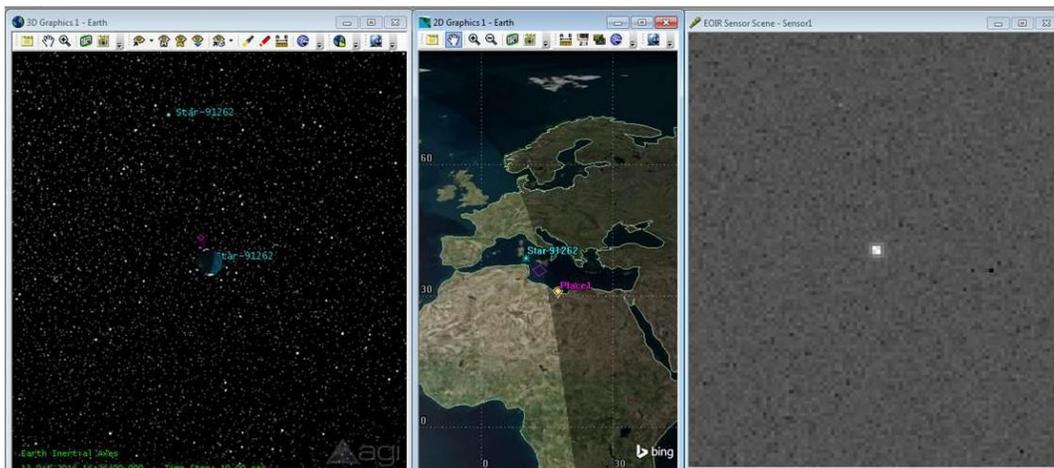
Figure 6 – On the left is the STK 3D view of Star-91262 (Orion) and the sensor on the earth, in the center is an overhead 2D view of the location of the sensor (Place1) and the projected star position, and on the right is the resulting image of the star

From this simulated image we can see Vega shining brightly in the center of our synthetic scene. Because we're using a simulator and have access to truth values as well and can look at the details to see what the in-band entrance aperture radiance value is for our star:
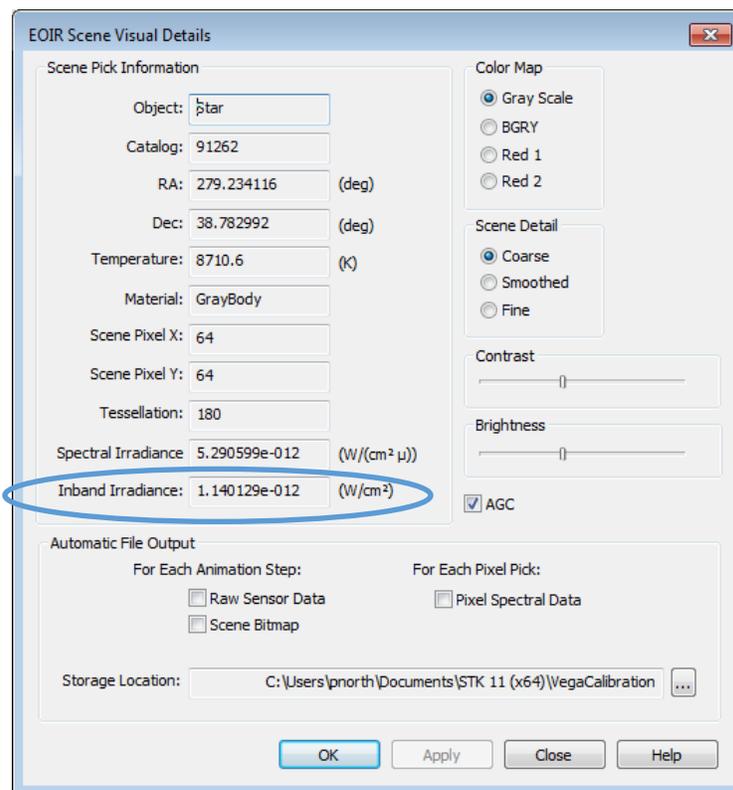
Figure 7 - Details page from the sensor synthetic scene in STKs EOIR.

For this particular sensor under these conditions the inband irradiance of Vega is E_{Ref} = 1.14x10^{-12} W/cm^2, so we can fill in our calibrated visual magnitude equation as.

$$Equation\ (6): v_{Mag} = -2.5\ \log_{10} \frac{E}{1.14x10^{-12}} + 0.03$$

However, this in-band irradiance is not readily available outside simulations unless our sensor is already well calibrated, that is knowing the *C2E* conversion value from Eq 3. Rather than trying to use the in-band irradiance estimates from a conversion in practice we would use the measured signal of the images.

Because the signal of the star is spread out over many pixels we want to sum up as much of the signal as possible to measure visual magnitude. However, our star of interest isn't the only potential source of signal, there may be other stars adjacent to our object of interest blurring extra signal in the direction of

our intended reference and the detectors will be noisy. We want to sum over the image region that is predominantly good signal from our target of interest. If we look at four row slices through the central portion of the raw data we can see that the majority of the signal here is contained in approximately the central 6-by-6 pixel region that we will sum over.
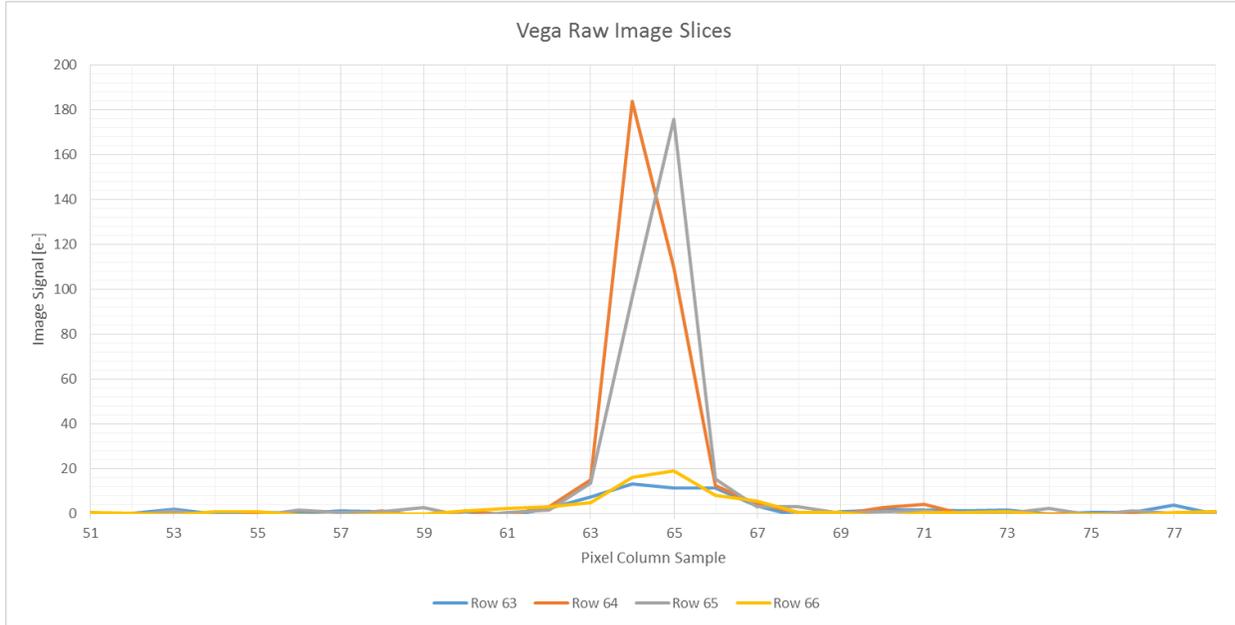
Figure 8 - Signal peak over the raw data file.

Loading up the central portion of the image in Excel and using a color scale for conditional formatting we can see the digital signal values in the image. I've drawn a 6x6 pixel box containing the values making up the majority of the signal for Vega which is equal to about 763 [electrons]. This image contains noise so it's not going to be perfect but it does give us a good measurement that will act as an initial estimate of the number of electrons from a known reference star.

	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78
51	-2.56352	0.800019	-2.19462	1.82592	1.25265	-1.20155	0.69827	-0.5991	-0.51301	1.04854	-0.01755	-0.88107	1.7317	1.62769	-0.64483	0.528325	1.03442	-0.44663	-1.13977	-0.23727	0.383304	0.679316	1.08434	0.722492	-1.63493	0.08707	-0.74612	1.6819
52	0.747145	-0.7392	-1.4698	0.860306	1.28529	-2.26451	1.5027	-0.93703	0.969391	2.66058	-0.96342	2.71754	3.74639	-1.16694	-1.27973	-0.92342	0.751152	0.275052	-1.01652	-0.64404	0.566788	-1.15441	1.79832	-0.82866	-1.19418	0.405145	0.242902	0.973804
53	-0.43321	0.61156	-0.53084	0.291579	-2.29093	-1.19075	-3.53433	-0.36495	-0.44288	-2.34495	-1.98114	2.04928	-1.34852	0.725467	-1.66568	1.96738	1.92126	-0.85279	-0.97032	0.929903	-0.50472	0.124068	0.475439	-1.33606	0.203928	-2.25372	-0.68029	-0.30411
54	-1.24946	-0.01546	-0.14893	-1.86895	-0.09386	-1.42007	-0.41806	0.790821	-3.21464	-0.45976	-0.43782	-0.20963	0.513919	0.861216	-0.11143	0.378315	1.00734	2.49038	-1.23025	-0.23463	-0.04798	-0.57736	-0.83614	1.44635	1.73285	0.700734	0.893727	0.058392
55	2.34528	-0.60732	0.136286	0.084553	1.49034	-0.56197	0.81381	-0.0875	0.117046	0.859638	-1.00729	-0.19549	0.186084	-0.40645	2.90285	1.34166	-1.39568	0.456342	-0.32655	-0.59885	0.088576	1.16698	1.58766	2.27363	2.38051	-0.51518	0.571916	0.921694
56	-0.68343	1.05859	0.312289	0.185906	1.1381	-2.84924	1.90773	1.08457	-1.21876	0.276455	1.00605	-0.88776	2.46225	-0.69298	-0.0157	1.48443	-0.75406	1.15785	-1.36477	0.907375	-1.30684	-1.22932	0.855653	-0.75539	0.736096	0.553093	-0.59669	-0.83759
57	-0.50524	2.74998	-1.17575	1.35109	1.68424	-0.29951	-0.23081	2.39123	0.928945	-0.9849	0.226386	1.23441	0.112747	1.55107	-1.32112	0.020803	-1.64828	0.051082	-0.14427	1.03154	-2.7287	0.155011	1.28705	1.64268	1.61401	1.13226	0.313681	0.232072
58	-0.81283	0.658295	-1.68589	-0.58903	1.09187	0.33792	0.540145	0.852644	0.861278	0.236594	1.82256	3.01451	-0.73686	3.34317	-1.98349	0.966489	-2.26859	-0.41198	0.420119	2.69753	-4.05986	0.415398	-2.93737	-1.34124	-0.50163	-0.24819	0.94613	1.09588
59	-0.46714	1.18805	-0.4579	2.99443	-0.09145	-0.15533	-2.03143	1.87869	-1.16128	-0.93516	-0.3425	1.28599	0.37436	-1.93129	-0.31123	-0.89668	0.426811	0.269751	-1.41847	-0.42866	0.47123	2.31048	0.534994	-2.83844	0.659972	1.01286	-0.96812	0.890361
60	-0.29098	-0.44182	0.800359	1.12725	-1.54294	0.599064	-1.69956	-0.73254	0.341472	1.32116	-1.4315	-0.61458	0.391715	1.06269	0.53514	0.119871	0.207656	-0.25637	1.31775	-0.0279	-0.30739	0.775136	-1.12952	0.919634	-1.03115	0.176144	-2.10882	0.457562
61	0.968841	0.358876	1.23017	-3.02979	2.48699	0.35313	-0.60959	1.26901	-1.40799	0.412436	0.620739	1.83281	-0.94496	2.84518	0.092033	-0.34075	3.59774	2.32635	0.711313	0.827851	-0.13577	1.16962	1.49305	0.520832	1.40241	1.94962	1.28709	0.103733
62	1.58371	1.70776	0.983572	-0.93924	2.06769	0.182802	-0.05659	0.442881	2.87745	0.494644	1.90602	0.541167	1.00526	1.56516	3.11141	1.5156	1.56006	0.29348	-1.15594	-2.7195	0.618603	0.746079	-1.70489	2.00013	0.856269	-2.10928	-0.4744	-0.59283
63	-1.62414	-0.08202	1.98236	-0.33668	0.4547	0.168179	1.08642	0.927885	-1.01069	0.641641	-1.04945	2.27874	7.39811	13.2752	11.3816	11.3227	3.36025	-1.77911	0.946017	1.6214	1.41157	1.15195	1.47368	-0.26041	0.548985	0.282391	3.58109	-0.37098
64	0.553654	-1.19824	-0.29076	-1.79363	-0.09343	-2.07216	-0.73962	1.0136	-1.95722	1.32336	-0.58927	2.8829	14.9527	183.735	109.655	12.3386	3.91796	0.461077	-0.21981	2.57541	4.19354	-0.58201	0.188894	-0.14868	-0.17896	0.181088	-0.30855	0.896501
65	0.039233	-0.7594	1.0715	-1.22146	-0.99545	1.5925	0.344688	0.872185	2.77917	-1.44804	0.524564	1.51321	13.6317	96.5937	175.9	15.3149	3.10616	3.08283	0.181139	0.672287	1.01551	-0.67708	0.211357	1.2257	-0.63847	1.33461	0.033131	0.519899
66	0.305449	0.150789	-0.67206	0.833664	0.908722	-0.61807	-1.08684	-0.27567	-0.39386	1.0973	2.24776	3.04153	4.90303	16.0761	19.1719	8.22087	5.38685	0.282262	0.502527	-1.29624	0.546777	0.58659	0.931602	-0.54875	-0.09125	-0.99791	0.420234	0.873596
67	1.43418	1.87813	0.902853	1.12387	0.264263	2.48188	1.46194	-2.44717	-1.97122	-1.16898	0.775022	0.021267	2.8213	3.74378	4.11776	0.875373	2.3894	-0.43606	0.861568	-1.00731	2.3723	0.599728	-0.49966	0.164664	0.902312	-0.5578	0.950347	-0.51485
68	0.512038	-0.31205	0.081898	-0.78299	0.134198	0.910761	-0.05601	-1.0135	1.48813	1.9869	-2.33325	1.83746	0.650085	-0.71191	0.926525	-0.40691	2.14196	0.522134	-0.42055	-0.39719	0.446986	-1.68426	1.58575	0.452192	2.65048	-0.37988	-1.52994	-0.87003
69	-1.04872	0.167946	-1.46521	2.0992	2.33061	0.163924	-1.08653	1.86951	1.74455	-0.38558	-0.54291	2.82548	1.71439	-0.07021	1.55486	-0.8892	0.812636	-1.04888	0.10518	1.00699	-0.35637	0.291182	0.294458	-3.54392	1.30707	0.568517	-0.50763	1.62793
70	2.14717	-0.25482	2.5368	0.448322	-1.27407	-3.33274	0.520746	-0.19542	-0.56356	1.38564	-1.31813	1.17626	0.856564	0.107849	-0.24981	1.80746	1.61036	-0.62014	2.01466	-1.63661	0.662862	0.547656	-0.00512	-2.30971	-0.32367	1.15528	0.582859	1.5692
71	-2.43924	-0.63586	1.46655	-0.76339	-0.54545	0.41694	-0.84866	0.303386	-0.62129	1.44305	1.30126	0.475782	0.335637	-0.14724	1.06523	1.73744	1.60086	-1.91927	-1.04403	-0.96953	1.07646	-0.153	0.934544	0.935384	1.02321	-3.13649	-2.33219	1.91731
72	-0.77148	3.46232	0.648048	0.97483	-0.26534	1.82509	1.53661	1.34719	0.529689	0.317033	2.10907	-0.1901	-0.34745	-1.53036	-0.48603	-1.54709	-1.17896	-1.27093	-0.74493	1.62871	1.65161	0.613749	0.099046	-0.63207	-1.795	0.52835	0.061212	
73	-0.78386	1.29355	1.04111	-1.23864	-2.21001	1.1437	2.64485	0.463036	-0.39154	-1.62072	-0.78717	-2.36061	-0.22868	-0.41968	0.060626	-1.77722	-0.47911	1.53381	0.896673	-0.51567	-0.08965	-0.3896	-1.0361	-0.39362	1.924	1.5986	0.44402	1.15367
74	1.01415	-2.00476	0.990682	1.41701	0.758725	-0.06727	-0.26571	0.689847	-0.69416	1.06964	0.301385	0.900992	2.06151	-2.90321	-0.20658	-0.99796	1.69102	0.946819	-0.12055	-0.65608	0.700939	-1.25644	0.907466	-0.99737	1.234	2.22876	0.750885	0.230852
75	0.607716	-0.64924	1.25668	2.06018	2.72477	1.27949	0.208263	1.6073	1.91447	1.61518	-0.55219	-0.5021	-1.25187	1.2553	-1.14661	-1.10573	1.41095	0.518629	1.08338	1.12504	-0.90016	-0.8774	2.08028	0.856238	0.538593	0.005717	-0.73934	-2.43682
76	-0.6302	-0.42506	1.82547	0.177394	0.637755	-1.50134	-1.3399	-0.16196	0.831961	0.879289	0.533318	0.271074	-0.90444	0.813106	-1.80936	-1.36993	-0.70848	0.997798	-0.29019	2.11153	0.207116	0.868174	-0.34782	2.16548	-0.48473	-2.69747	-0.53819	3.33883
77	0.110916	0.117913	0.640471	-1.10095	-0.65986	1.11338	-0.30712	-0.77725	-0.98311	-0.15041	-0.45867	-1.11066	1.76723	-0.00379	0.274445	-1.25578	1.61652	1.16631	-0.12967	0.907646	-0.13357	-1.03668	-1.43736	-1.59165	-1.88283	0.453519	1.209	-0.62828
78	-0.52905	-0.3768	-0.06672	-1.43712	0.262707	-2.39597	-0.15861	-1.60631	-3.18	1.60298	-1.00924	-0.61916	-1.01223	1.76282	-0.39953	2.06622	-2.11513	1.18709	-0.05773	1.13467	-0.02566	0.159552	-0.02092	0.675041	-1.34176	1.88357	-0.0282	-0.39286

Figure 9 - Data peak is colorized and the region in the rectangle will be summed up to represent the summed counts of the signal source.

With the summed counts of Vega we can estimate the $C2E$ sensor calibration variable as well by rewriting the above visual magnitude equation (Eq 1) with the conversion of the C2E sensor calibrations (Eq 3) as demonstrated in Eq 4:

$$Equation\ (7)\ v_{Mag} = -2.5\ \log_{10} \frac{S}{763} + 0.03$$

A final note is that we need to be certain to include consideration for the background and viewing conditions. In the simulation we can simply turn off the atmosphere so we don't have any ambient illumination or path transmission, and we can ignore the negligible contributions of galactic background and starlight with such a bright star as our reference. In practice we would want to subtract the background value out and account for transmission path differences with the sensor calibration process when necessary.

Method 2 - In-Scene Image Calibration: *What if we don't have Vega?*

We don't have to use Vega or another incredibly bright well calibrated celestial target. In cases where we may not have an ideal calibration reference near our intended target object we can repeat the steps with another star that does happens to be within our field of view or at least nearby our object of interest. For example, later on we'll examine the Orion constellation and use the brightest star in the region, which happens to be Rigel with a v_{Mag} = 0.18. This is a very bright star within a distinct constellation and we chose this example because we wanted to use a well-known and easily identifiable constellation with both synthetic and real image examples so that you could recreate this analysis using STK with EOIR and a digital camera in your own backyard.
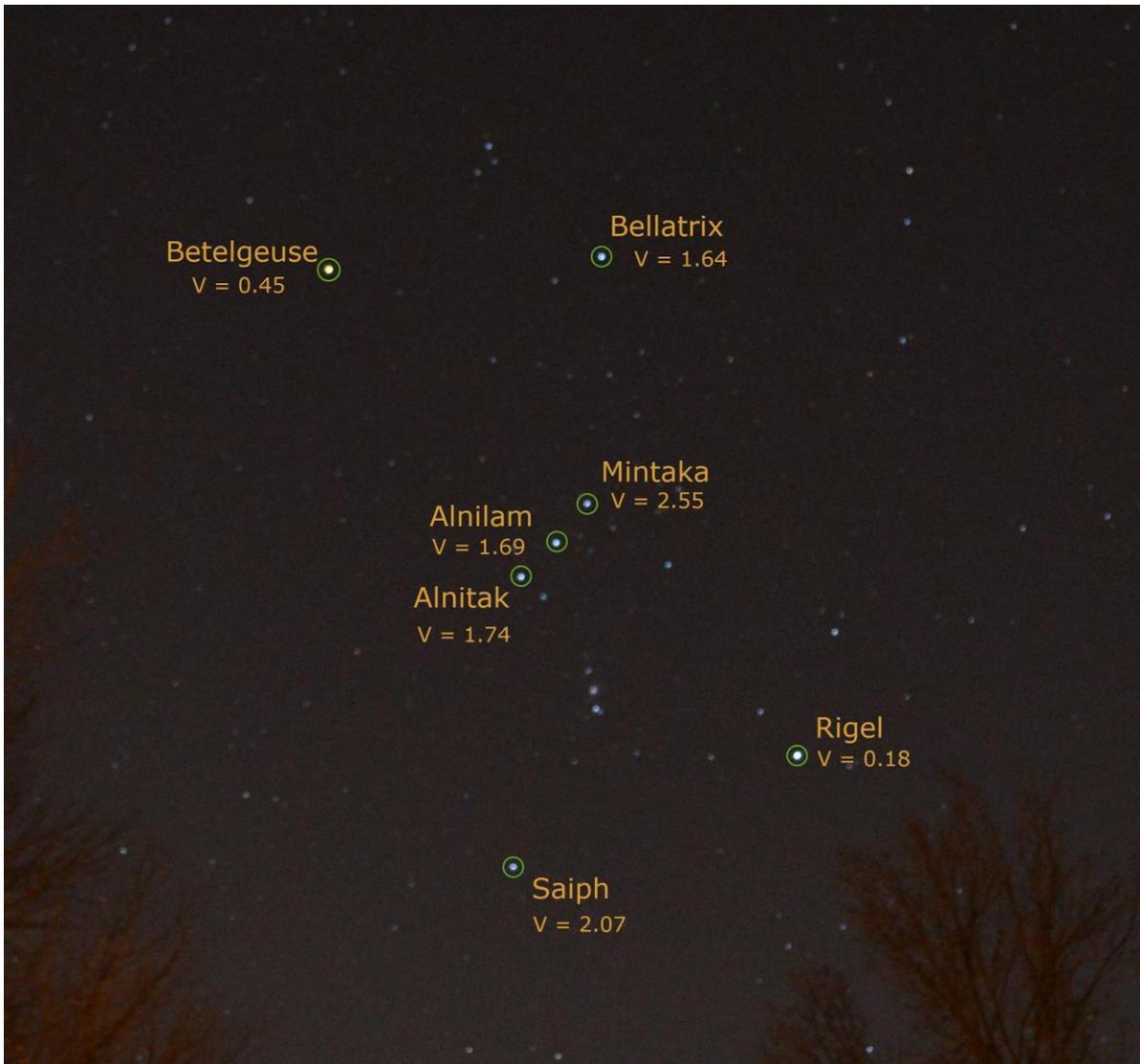
Figure 10 - Orion constellation with the seven brightest stars labeled by name and cataloged Apparent Magnitudes.

Looking back at our equations for v_{Mag}, we need to establish a value for v_{Ref} , s_{Ref} , and E_{Ref} . The v_{Ref} value is easy, since it is the cataloged visual magnitude for our reference star. Once we have established the v_{Ref} value, we will use an EOIR sensor to measure the E_{Ref} and s_{Ref} terms.
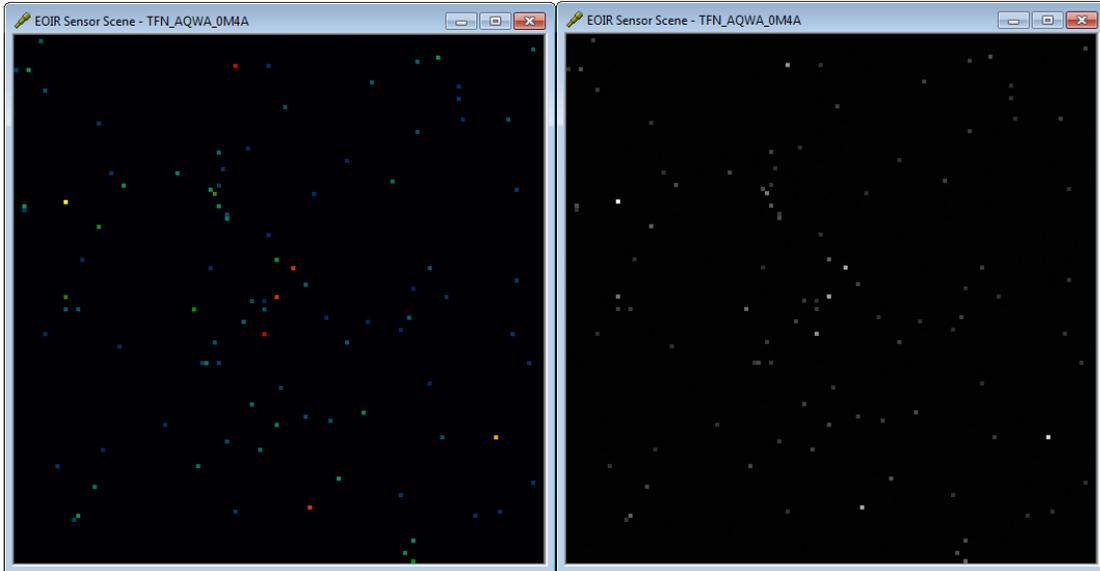
Figure 11 & 12 - Here we can view the scene in a colorized image on the left or good old gray scale image on the right. These images are 10° x 10° FOV and show us the entire constellation.

The sensor we'll be simulating in this STK scenario is using the default EOIR settings with the only change made to the Field of View (FOV). We focused this specific analysis on the visual band from 0.40 to 0.70 microns with a flat spectral response.

Sensor Settings	
Sensor Type	EOIR
Field of View	10° x 10° for overviews, 1° x 1° for individual stars
Number of Pixels	128.00 x 128.00
Spectral Waveband	0.40 um – 0.70 um
F/#	2.00
Effective Focal Length	11.00 cm
Diffraction Wavelength	Band Center
Image Quality	Diffraction Limited

Table 2 - EOIR Settings for the analysis.

As shown below in Fig. 13, we have the option to either look at the "Details…" of the star (which will give us the E_{Ref} value) or to Save Raw Sensor Data from the image (which will allow us to calculate s_{Ref}). When we pull the raw data as the image we sum the count values in a six-by-six pixel grid around the center of the point source as we did earlier with Vega.
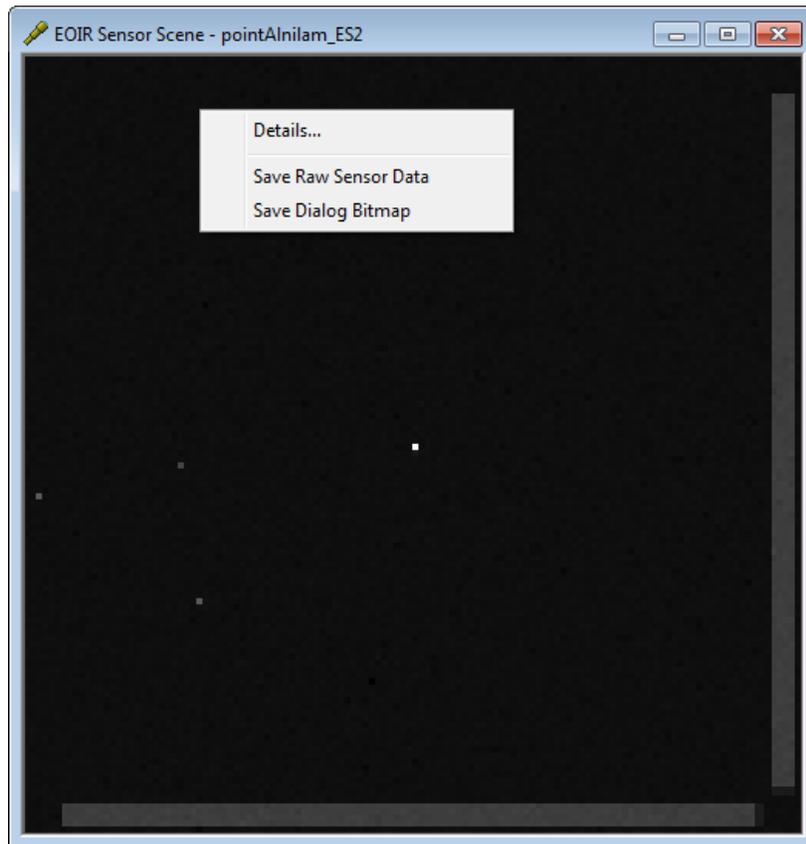
Figure 13 - Using the STK Pointing Property for our sensor we targeted Alnilam in the Orion constellation. In this image we are looking at a 1° x 1° FOV.

Method 3 - Multiple Reference In-Scene Cal: *Fitting to all of your available stars*

If we don't have an ideal calibration reference or a particularly good alternative we can use multiple less than ideal reference stars to create a curve fit in the form of our visual magnitude equation, essentially solving for a log-fit. Below is a sample real image taken where the cyan circles are around bright objects detected in the sky above a relatively high threshold and the red asterisks are correlated with bright stars having a visual magnitudes of 6.0 or brighter. An additional note is the two very bright objects at the bottom center of the image are planets.
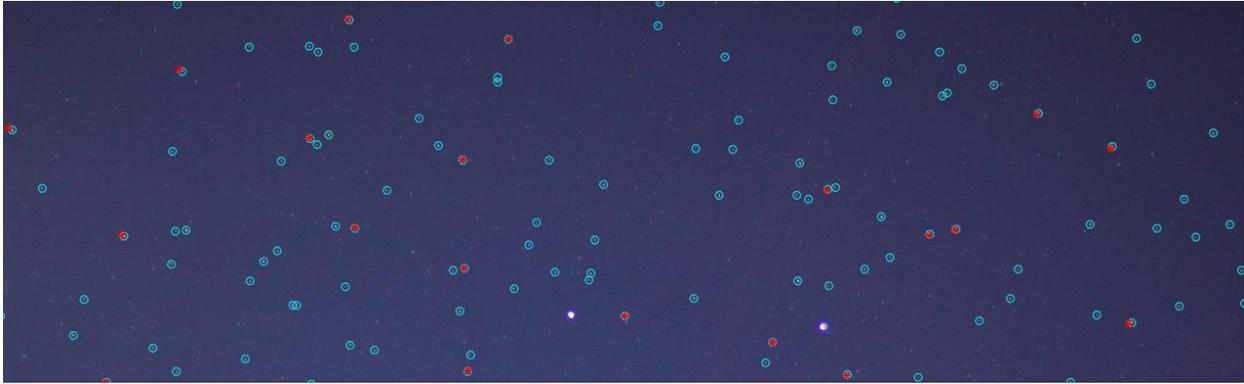
Figure 14 - Real image of the night sky with signal sources highlighted.

Once we have a correlation between stars in the image and a catalog we can plot the summed counts as we've discussed against the catalog visual magnitude values for those same stars. This sample data is plotted below as a vmag-vs-counts scatter plot with a curve fit. This gives us an idea of the variability of the measurements as well as a visual way to map summed signal measurements to visual magnitude.
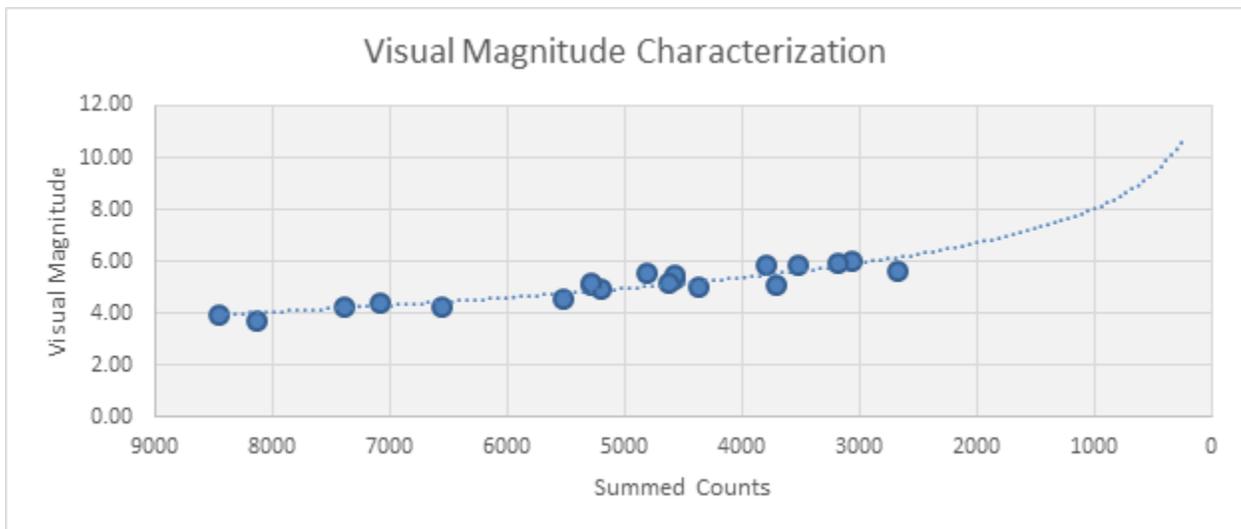
Figure 15 - v_{Mag} curve using the summed counts in the calculation.

As shown in the figure, we typically plot the summed counts in reverse order so that the curve is going from the lower left to the upper right and steadily increasing. From this plot and the curve fit to the data we can see a v_{Mag} of 4 corresponds roughly to 8,000 summed counts, a v_{Mag} of 6 corresponds roughly to 3,000 summed counts, and if we extrapolate from the data and follow along the curve then a v_{Mag} of 8 corresponds roughly to 1,000 summed counts. We can also see that there is variation at each of these known points from the curve fit. This can be due to noise, local background conditions, differences in atmospheric path, or color-temperature of the stars themselves. However, even with all of these variations the fit to the data is still excellent and typically with bright stars this is as we would expect.

A final note is that the vmag-vs-counts plots gives us a very good qualitative feeling for our sensor systems detection limit. We can see that as our total counts approach zero on the right side of the graph the visual magnitude curve starts climbing rapidly and as we approach the noise limit. The uncertainty of these smaller signals will result in very large variations in any visual magnitude estimates we try to make.

Estimating Vmag: *Walking through an Orion example*

Once we've established the method of calibration, we can begin by choosing our reference star or stars. Since in practice we will only have images to work with we'll start by establishing the measured signal of our references. In this example we'll look at the Orion constellation in both real and synthetic imagery and will measure the signal of the brightest star in the FOV, the star Rigel. From the catalog we found the visual magnitude of Rigel to be 0.18 (v_{Ref}= 0.18 for our example). We can now plug this value into Eq 1 with E_{Ref} or Eq 5 with s_{Ref} , and then solve for the v_{Mag} of any other stars or objects in the images. We also include calibrating measurements of Vega as well to compare.

With the inherent noise of the reference measurements we also want to verify how the summed signal s_{Ref} varies. To account for this, we took six measurements at 1.0 second increments of our reference star Rigel. This small selection gave us an average calibrated value for the s_{Ref} rather than a single noisy measurement. The value of E_{Ref} will not waver over the selected time span because it is a truth value only available in simulations so we can simply use the E_{Ref} value we read off of the details page and plug that into Eq 1 for academic purposes. Once our s_{Ref} has been measured we will plug it into Eq 5 of the v_{Mag} calculation and estimate the visual magnitude for some of the other stars in the Orion constellation. The results are shown below: using the in-band irradiance E_{Ref} in figure 16, summed
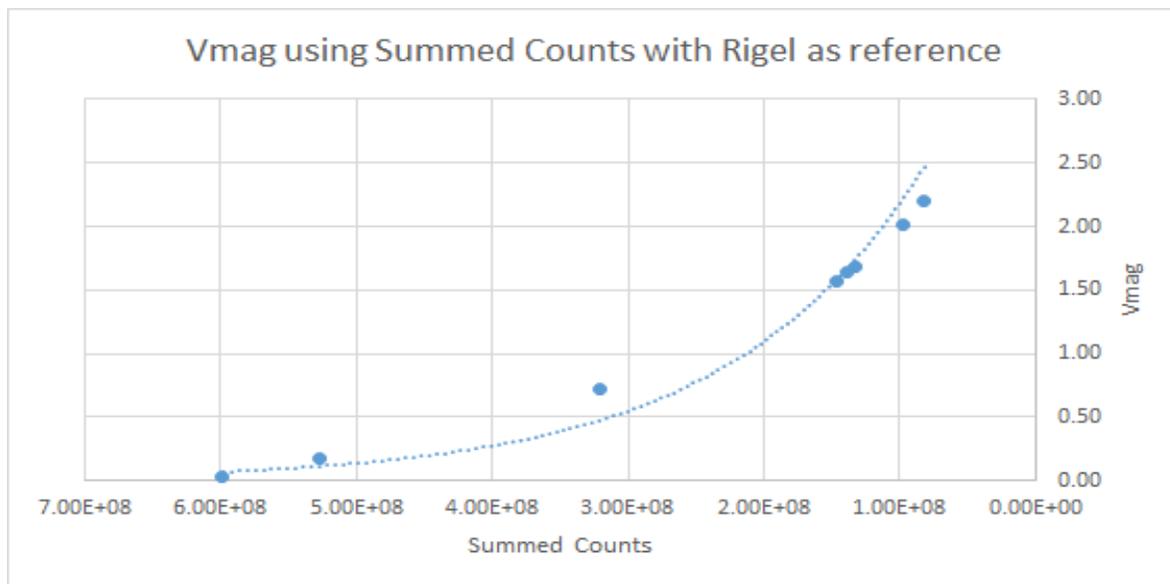
counts s_{Ref} in 17, and the numerical values summarized in table 3.

Figure 17 - Results of v_{Mag} using the summed counts of Rigel. Each point refers to a measured star.

STAR	Cataloged Vmag	Inband Irradiance	Vmag	% v_{Mag} Error	Summed Counts	Vmag	% v_{Mag} Error
Alnilam	1.69	2.6233E-13	1.6352	5.18%	1.3803E+08	1.6351	5.19%
Alnitak	1.74	2.5179E-13	1.6797	5.71%	1.3246E+08	1.6797	5.71%
Bellatrix	1.64	2.7842E-13	1.5705	6.61%	1.4648E+08	1.5706	6.60%
Betelgeuse	0.45	6.0914E-13	0.7205	-22.05%	3.2046E+08	0.7205	-22.06%
Mintaka	2.25	1.5615E-13	2.1984	4.86%	8.2145E+07	2.1985	4.85%
Rigel	0.18	1.0021E-12	0.1800	0.00%	5.2722E+08	0.1800	0.00%
Saiph	2.07	1.8387E-13	2.0210	4.62%	9.6740E+07	2.0210	4.62%
Vega	0.03	1.1401E-12	0.0399	-0.91%	5.9983E+08	0.0399	-0.91%

Table 3 - Summary of calculations using Rigel as our reference star. Cataloged v_{Mag} values are sourced from the HYG database.

Both the irradiance and summed counts methods of analysis show a strong trend between the cataloged measurements and the results of this analysis and to each other. We can compare each method of calculation side by side, as shown in the table above to see the inherent variability from the true irradiance to measured signal. The summary of the results also show us a strong trend between the values that had been calculated and the expected catalog values.

When we're looking at the data it is important to get perspective. One way to view this visual magnitude data is by directly comparing the measured data and the cataloged data. Here we see a strong trend between our results which will show us the accuracy of our measurements.
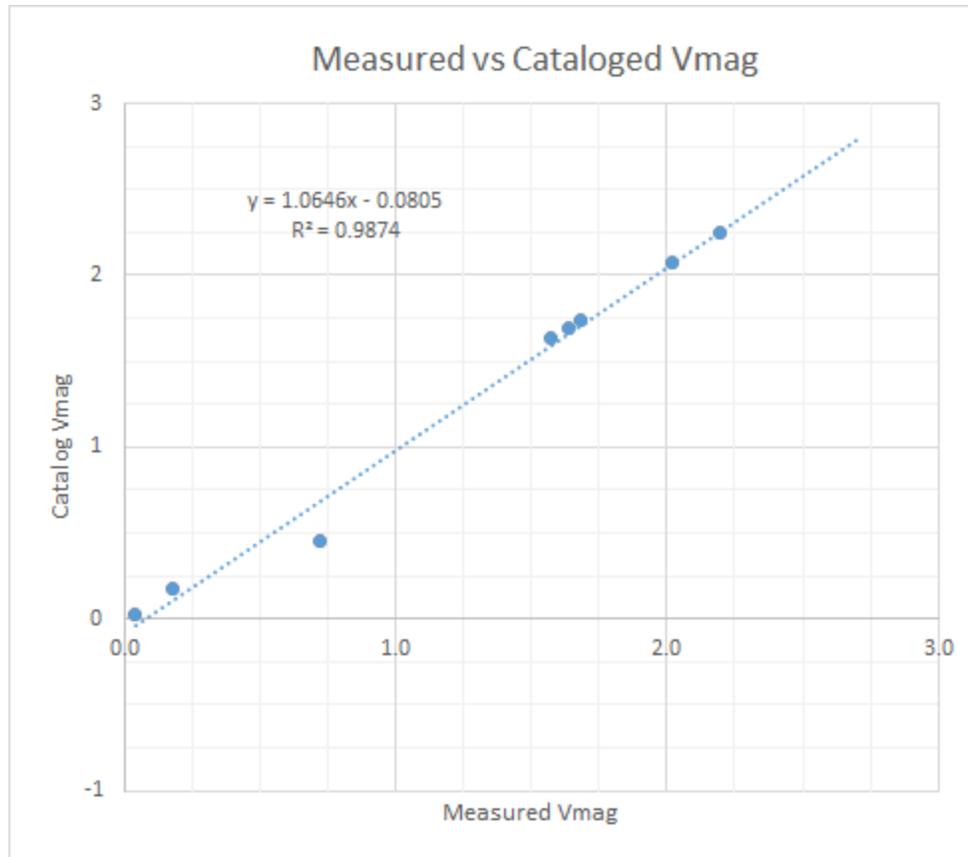
Figure 18 - Comparison of measured results using Inband Irradiance or digital counts. Each point refers to a star that was measured.

Our calculated v_{Mag} terms are strongly correlated with the cataloged values. We have a solid trend between our values and catalog, with a slope, m = 1.06. We have also calculated the error between our results. The obvious outlier with the largest error is Betelgeuse. Once again, we are measuring a spectral band between 0.40 um - 0.70 um and Betelgeuse is a red variable star. The other variable stars are Alnilam and Vega. In this setup, Vega was the only main sequence star, whereas the other stars were giants or supergiants. Looking into the data, it is likely that the differences in our calculated values and the cataloged values is due to the color temperature and variable nature of the stars.

When Things Get Complicated

So far, we have been looking at best-cases scenarios for measuring visual magnitude to emphasize the concepts in general. However, it's worthwhile to consider some of the complications that you would run into in practice. We will briefly talk about the following four considerations:

- Relative Motion

- Atmosphere

- Spectra

- Pixel Sampling and Noise

Relative Motion - What if our star is streaked out?

When we're staring at the stars it's hard to perceive how fast they're moving across the sky, but remember that our planet is rotating at about 360-degrees every 24-hours, or roughly 72.7 microradians per second. Imaging the stars through a telescope even for a fraction of a second will result in relative motion if they're not being actively tracked.

Figure 19 - Apparent motion of stars over the course of an evening. Streaks are due to the rotation of the Earth and the latitude of the observer.

Any relative motion of the stars within the telescope's field of view causes the stars to appear as streaks rather than points. We can still follow the same procedures we've already outlined, but instead of summing over the center point we will sum the over the entire streaked signal to find the s_{Ref} term. The remaining analysis will be the same.

Figure 20 - Real image of the Orion Constellation with motion blur.

In practice, star streaks greatly increase complexity. We have greater chances of stars overlapping each other. This in turn leads to a more complicated background, because our signal is distributed over more pixel samples and the noise contribution from each of those samples is accumulated into the summed signal and lowers our signal-to-noise ratio. This aspect will be described further in the sampling and noise section.

Atmosphere - Blurring the lines

Even if the stars are being perfectly tracked, the presence of a fluid atmosphere in between the sensor and the star introduces some amount of blur. In addition, the variation of moving pockets of hot and cold air causing random temperature fluctuations throughout the atmosphere is known as turbulence and the small particles in the atmosphere that absorb and scatter light in random directions over broad areas is known as turbidity. Both of these effects appear as blurring in an image and reduce the contrast.
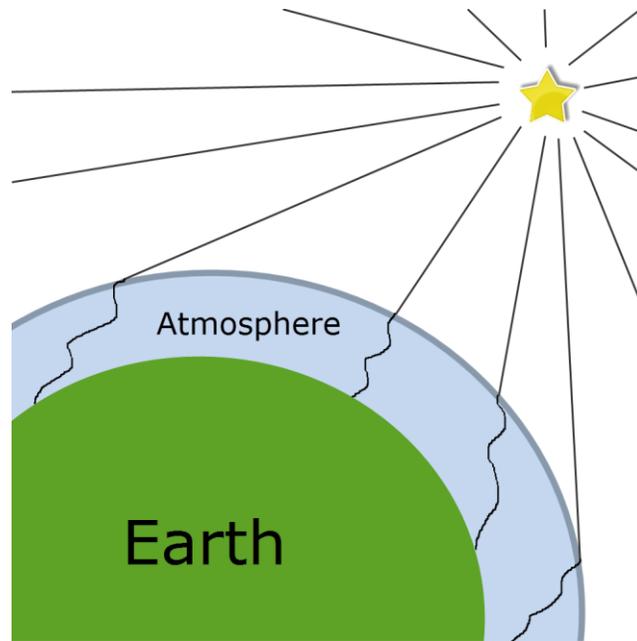
Figure 21 - Representation of the atmospheric effects on stellar signals as they travel through the atmosphere.

Photons traveling along a path through the atmosphere can be scattered out of the path or completely absorbed by particles in the atmosphere resulting in a loss of signal. This is known as the atmospheric transmission and is a ratio of how many photons arrive at the sensor relative to the total number of exo-atmospheric photons that we started with.

The background of the night sky isn't perfectly dark either, since any light from the ground, the Moon, or even the twilight effect of the Sun below the Earth's horizon can be can be scattered into the path of the sensor. This is typically referred to as the background or path radiance in an image. This reduces the contrast of any stars in the image and adds an additional amount of noise. A final source of path radiance is any photons that might be thermally emitted by the atmospheric particles themselves. This would be insignificant in the visible spectrum but is present in the thermal infrared portion of the electromagnetic spectrum.

Finally different paths will have different properties, so calibration is only applicable under the same or very similar conditions. Alternatively if you have a good model for estimating the atmospheric properties then applying an atmospheric correction would help overcome this issue.

Spectra - Wavelengths and colors

The EOIR sensor we used in the earlier examples measured light in the visible waveband from 0.40 um – 0.7um to correspond relatively well to the human visual system response and the visual magnitude metric. However, as we saw earlier, in Figure 2, each star temperature has a full-emission spectrum that goes beyond the visible spectrum and the visual magnitude metric. We have sensors that can optically measure multiple portions of the electromagnetic spectrum from the ultraviolet to the infrared in

bands, this lets us sample the spectrum of our scenes. For the focus of this analysis we limited our observations to the visible waveband and this affects our measurements because each star is a different temperature and even then some stars are variable stars (meaning their luminosity varies over time) and can peak at different wavelengths and magnitudes at different times. When we focus on the visible portion we are excluding any data outside of the narrow visible limits as well as a variations within. This makes the visual magnitude metric itself limited because it is for a single very specific spectral response.

We can spectrally calibrate a sensor with a different bandpass to visual magnitude, but that introduces a further problem with the in-band signal and its correlation to the visible spectrum. If the object of interest's spectrum has little to no overlap with the visible spectrum then using visual magnitude is a poor metric. Similarly if the object of interest and the reference used to calibrate the visual magnitude equation have dissimilar spectra then the estimated visual magnitude will be a poor estimate. However, calibrating to target irradiance with a reference to the spectral band and applying a visual magnitude correction based on the known object spectra and sensor response, or a color correction for catalog stars, can still be useful knowing these limitations.

Pixel Sampling and Noise - Optimizing signal-to-noise

In the examples we've shown there has always been plenty of signal available from our target to simply sum everything within a fairly large region of interest. But in a more complicated background or for a lower total signal it may become necessary to perform some basic target detection and perhaps even enhancement or restoration. The spatial distribution of signal from a point source target will be in a specific pattern called a Point Spread Function or PSF. This PSF is a function of the optics, the shape of the detectors, relative motion blur, and atmospheric blur. When normalized to unity, the shape of this PSF will often have a well-defined peak and much lower tails. Depending on the amount of blur, the bulk of the signal could be contained in a few pixel samples or could be spread out across hundreds of samples. For isolated bright targets it's sufficient to sum everything up in a large area, however, for lower-contrast targets it makes more sense to sample the proportionally strongest signal samples rather than all of them. For example if the peak of our PSF is 40% of the total signal but the tails contain less than 0.1 % of the total signal the relative SNR falls off drastically at the tails and they offer very little value.
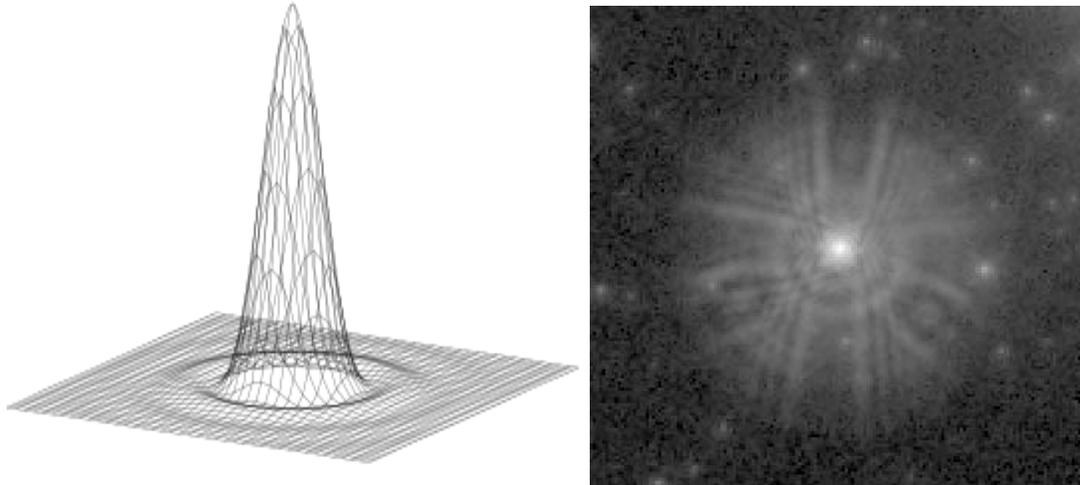
Figure 22 - a. Airy Function b. The point spread function of Hubble Space Telescope's WFPC camera before corrections were applied to its optical system.

If you know the PSF or can make a pretty good guess at it then you don't need to exhaustively sample all of the pixels but can selectively choose the strongest SNR samples to measure. One simple way to do this is to draw a threshold and only choose samples above the threshold to measure and estimate the total signal of the target. This is analogous to estimating the total mass of an iceberg by measuring the amount of ice floating above the surface. Because ice has a known density and the amount of ice above the surface of the water is proportional to the total mass of the iceberg you can get a very good estimate from a limited sample. Similarly by setting a threshold with knowledge of the PSF you can measure just what's above the threshold and use that to estimate the entire sum of the target.
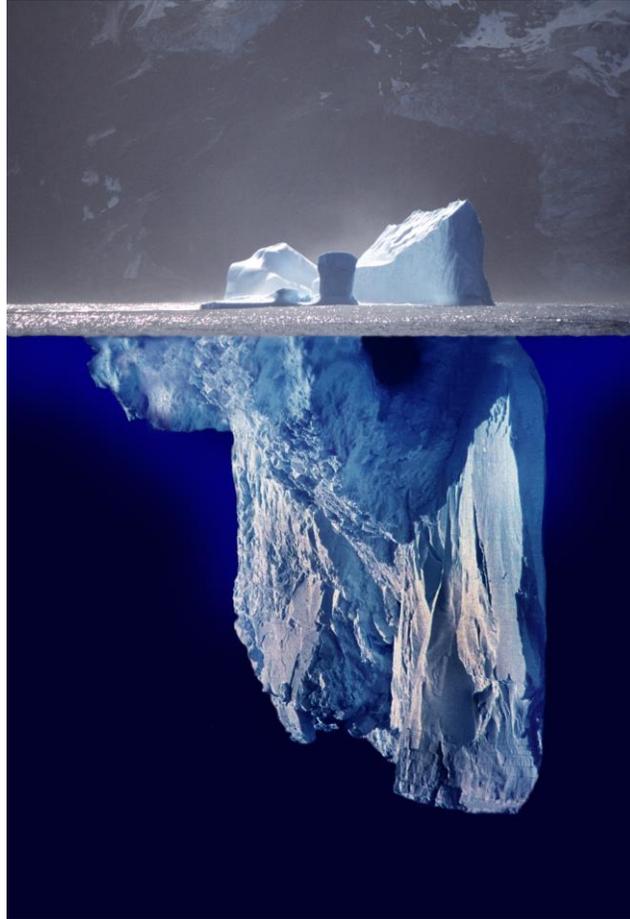
Figure 23 - Iceberg example

An alternative is to measure the object of interest using the same mask as the calibration reference. This way they will both have roughly the same percentage of total signal and can be directly compared in the relative definition of irradiance or signal of visual magnitude. Because every sample contains noise, there will always be level of uncertainty in the measurements. Also because the conditions of the measurements themselves can have different levels of noise it's important to try to quantify the amount of uncertainty in them. On a log scale such as with visual magnitude, a percent error translates to a simple plus or minus. The uncertainty bounds are especially important to note for time-varying targets so that one can differentiate true signal variations from measurement noise. Also you can see the confidence bounds of the measurements as well so that variations clearly stand out and can be differentiated against random noise. Finally summing over more samples even with an isolated object of interest does not necessarily improve SNR, total target signal may increase with each additional sample but so does the cumulative noise shown in the table below with a few different noise levels and numbers of samples. If the addition of proportionally lower signal samples does not significantly outweigh the additional noise then fewer strong samples may be optimal.

RMS Noise of Summed Samples

		RMS Noise per Individual Sample
		0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
Number of Samples	1	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
	4	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
	9	0.75	1.50	2.25	3.00	3.75	4.50	5.25	6.00	6.75	7.50
	16	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
	25	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00	11.25	12.50
	36	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50	15.00
	49	1.75	3.50	5.25	7.00	8.75	10.50	12.25	14.00	15.75	17.50
	64	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00
	81	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
	100	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00

Figure 23 - Summary of RMS Noise

Common Variations of Vmag for Satellites

Visual magnitude can be a tricky metric to use for satellites because it was initially meant to measure the brightness of stars. However, now with Space Situational Awareness (SSA) applications looking at the brightness of satellites and space debris and trying to measure their brightness in visual magnitude units a number of problems have arisen. First the conditions of the measurement can drastically change the measured visual magnitude. For star targets we have perfect point sources that are essentially isotropic emitters at a constant range as we mentioned before. For satellites we have complex geometric shapes, unique material surfaces, and variable orientation angles, illumination angles, and ranges. A number of conventions can be made to try and normalize the disparate conditions and make apples-to-apples comparisons such as:

- Apply atmospheric correction

- Scale to a standard reference range

- Scale to a standard reference illumination angle

The good news is that because the primary illumination source for satellites is our sun we know what the illumination spectra is and it fits the visual curve very well. When calibrating satellites to known stars it is ideal to apply color correction to the visual magnitude as well.

Summary

The visual magnitude metric gives us a way we can quantitatively measure the magnitude of stars, and we've show examples of this using STK with EOIR. In this analysis we've demonstrated three ways that could be done; using a standard calibration star, using a bright star near our object of interest, and using multiple less-bright in-scene sources. Using each of these methods we have to consider the unique factors that will affect our results as well as the complications and variations that may be applicable. Throughout this analysis we've confirmed the quality of the measured data with STK and EOIR for stars,

however, we can take the same steps of this analysis and apply it to other studies, such as measuring the visual magnitude of satellite objects.

What's next?

We hope you have you enjoyed this study. Are you interested in learning more about EOIR? Visit our [site](#) to watch videos, read about the latest new features, and find out more. If you're curious about seeing EOIR in action you can watch a demonstration of its capabilities on our blog, [Missile Defense](#). Have any questions about this analysis or EOIR, or would you like to see a follow up focusing on simulating satellites and measuring their visual magnitude? Then head over to our [support department](#), email us at support@agi.com, or give us a call at 1.610.981.8888 or 1.800.924.7244!